Lattice

Complete Summary

Part – 2

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GATE CSE AIR 53; AIR 67; AIR 107; AIR 206

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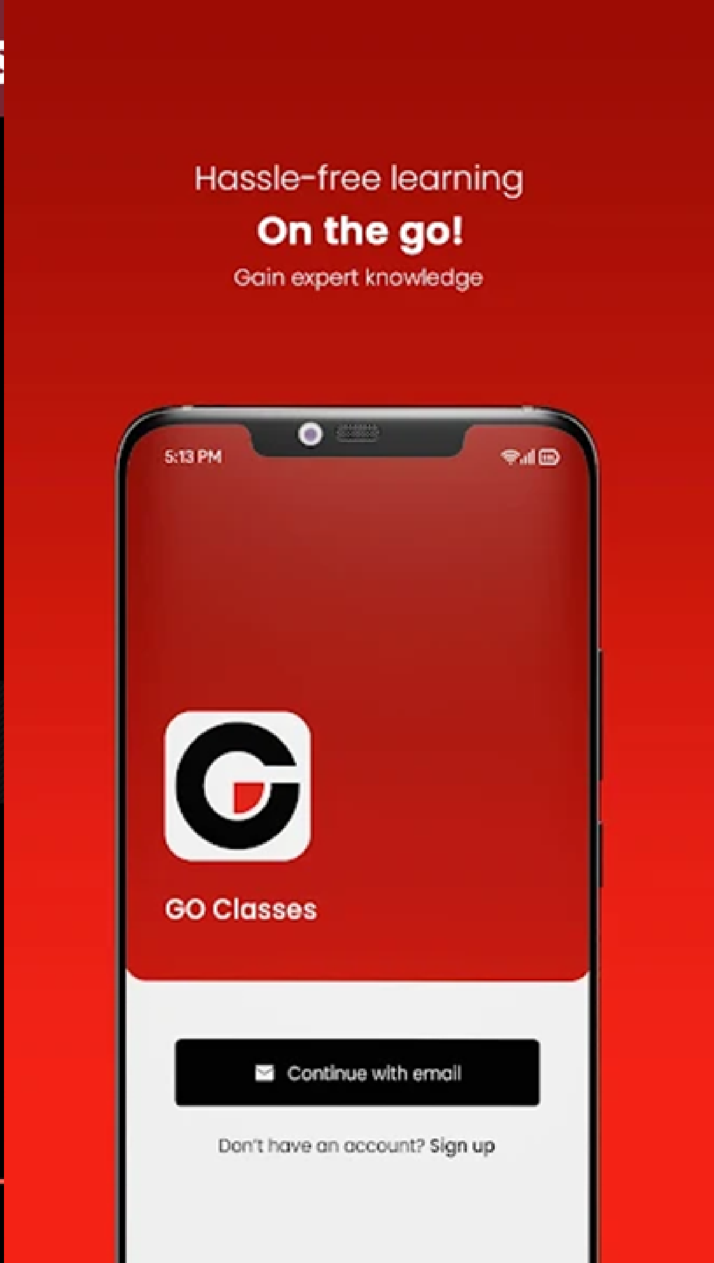

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Lattice

Complete Summary

Part – 2

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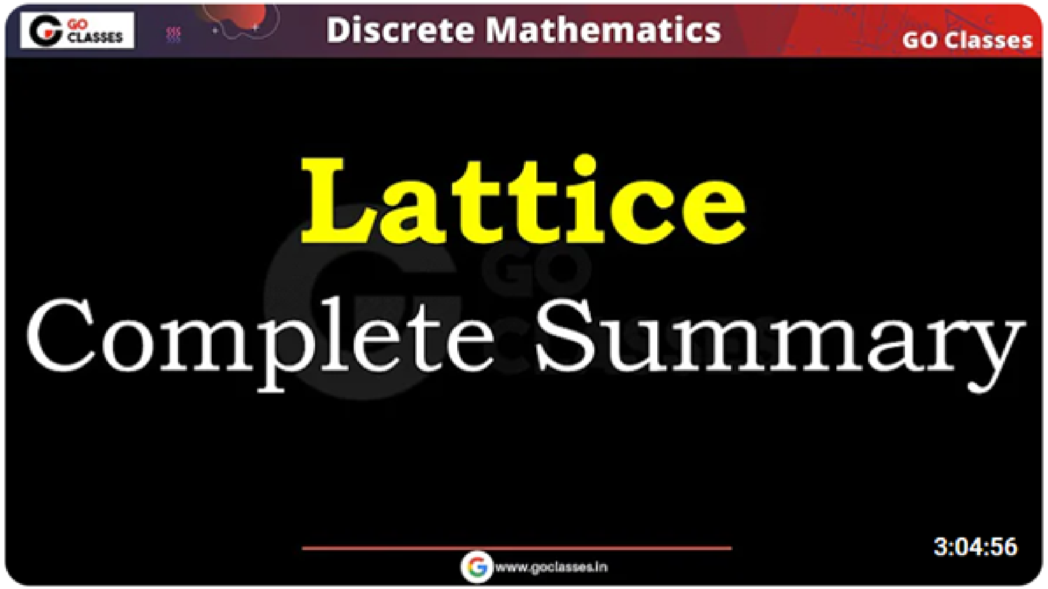

Lattice - Complete Summary - Part 1 | POSET Lattice | Set Theory | Discrete Mathematics | With NOTES

75 views • 12 hours ago

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Discrete Mathematics Complete Revision, **Summary** & GATE PYQs: ...

New

Link in the Description & Pinned Comment.

Boolean Lattice
or
Boolean Algebra

Definition

A Boolean algebra is a lattice with 0 and 1 that is distributive and complemented.

Bounded

Least

Greatest

Example

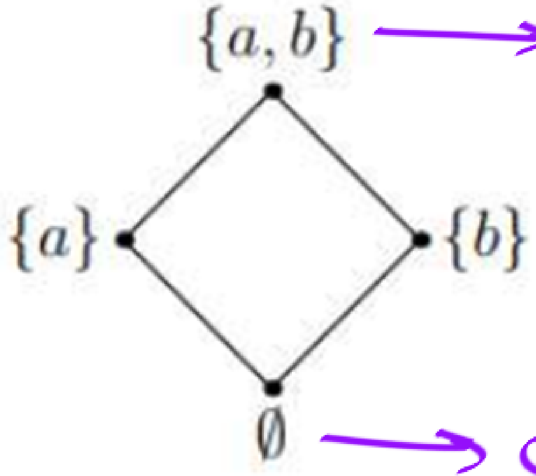
Let A be a set. Then $\langle \mathrm{pow}(A), \subseteq \rangle$ is a Boolean algebra.

$A = \{a, b\}$

$(P(A), \subseteq)$

Boolean Algebra

- Bounded
- Distributive
- complemented



$\{a,b\} \rightarrow 1$

$\emptyset \rightarrow 0$

Figure: $A = \{a, b\}$.

$\overline{\{b\}} = \{a\}$

$\overline{\phi} = \{a, b\}$

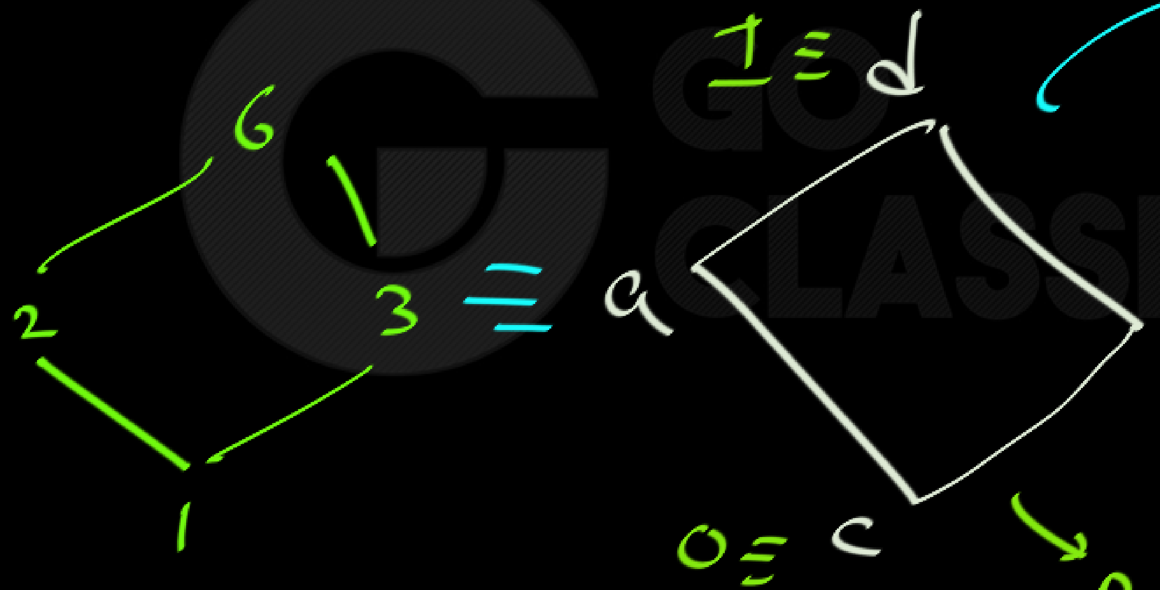
$\overline{\{a,b\}} = \phi$

$\overline{\{a\}} = \{b\}$

Example

$\langle \{1, 2, 3, 6\}, | \rangle$ is a Boolean algebra. ✓

Divides



Distributive ✓

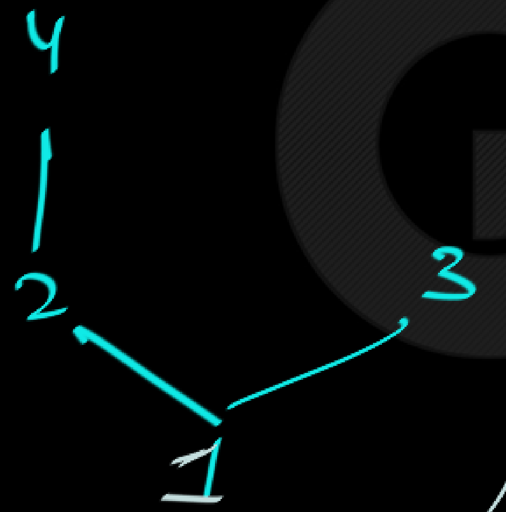
Complemented ✓

Bounded

$$\left\{ \begin{array}{l} \bar{a} = b \\ \bar{b} = a \\ \bar{c} = d \\ \bar{d} = c \end{array} \right\}$$

$(\{1,2,3,4\}, \mid)$ is Boolean Lattice? No

Divides



Not Even a Lattice

$3 \vee 4 = LUB\{3,4\} = DNE$

POSET

$(\{1,2,4,8\}, \mid)$ is Boolean Lattice? No

Greatest ← 8

4

2

Least ← 1

TOSET → Lattice ✓

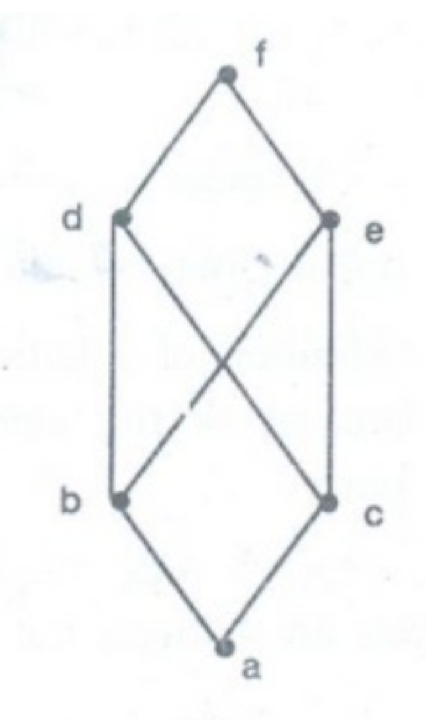
TOSET → Distributive Lattice

Bounded ✓

Distributive ✓

NOT Complemented

$\overline{2} = DNE$; $\overline{4} = DNE$; $\overline{1} = 8$; $\overline{8} = 1$



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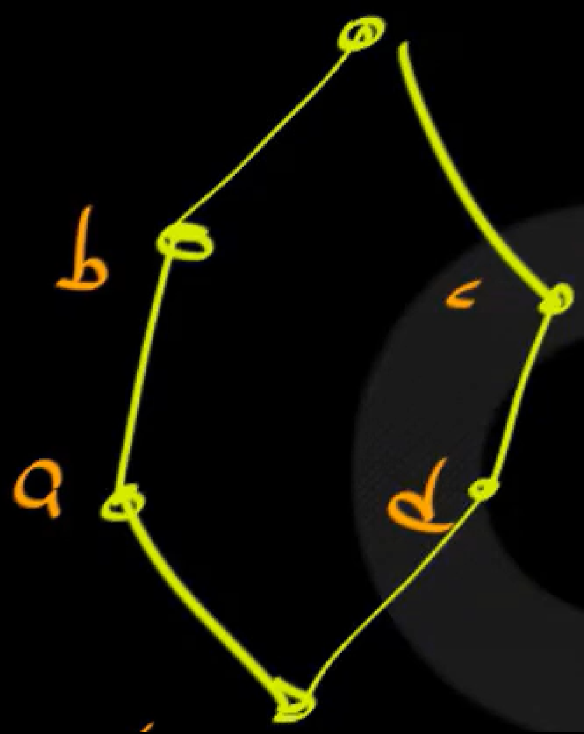
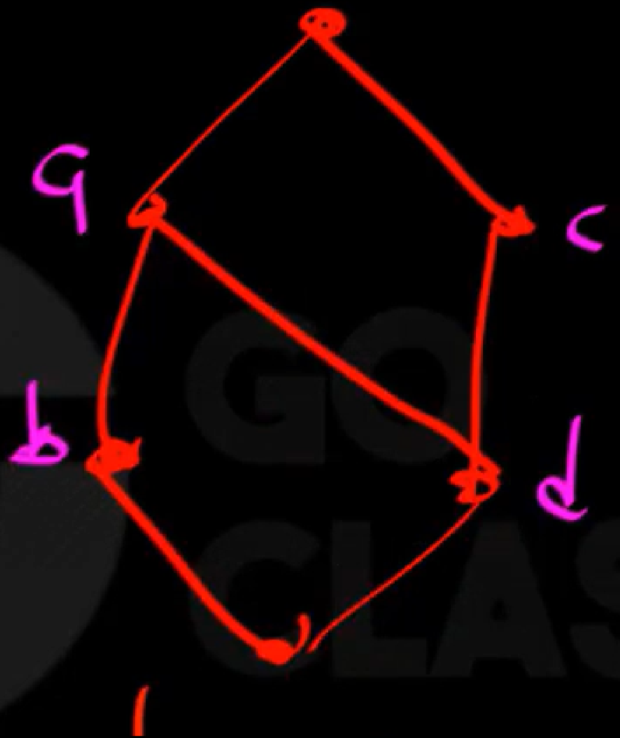
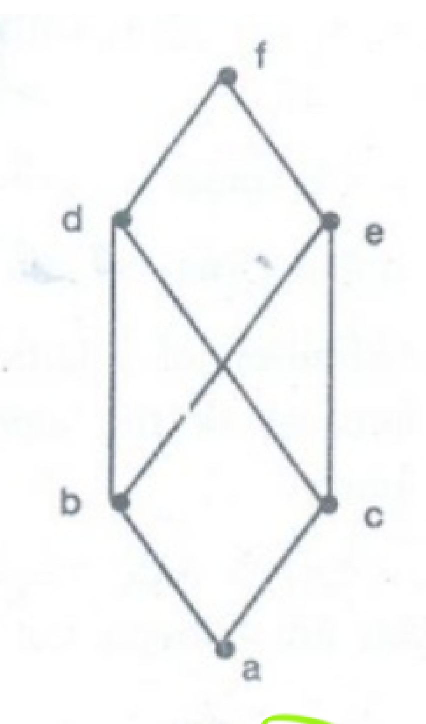




Which is a Boolean Algebra? None



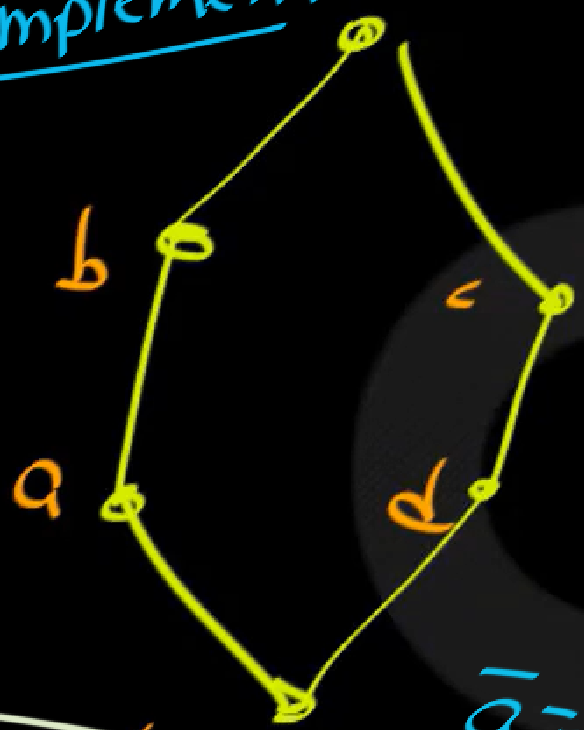
(b)

Not even a Lattice

$b \vee c = DNE$

$d \wedge e = DNE$

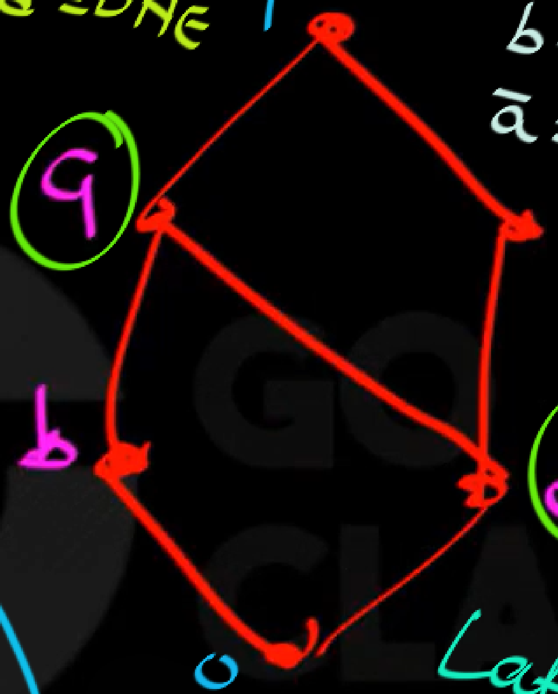
Complementes ✓



$\bar{a} = c, d$

NOT Distributive

$\bar{d} = DNE$



$\bar{b} = c$

$\bar{a} = DNE$

Lattice ✓

Bounded ✓

NOT Complemented

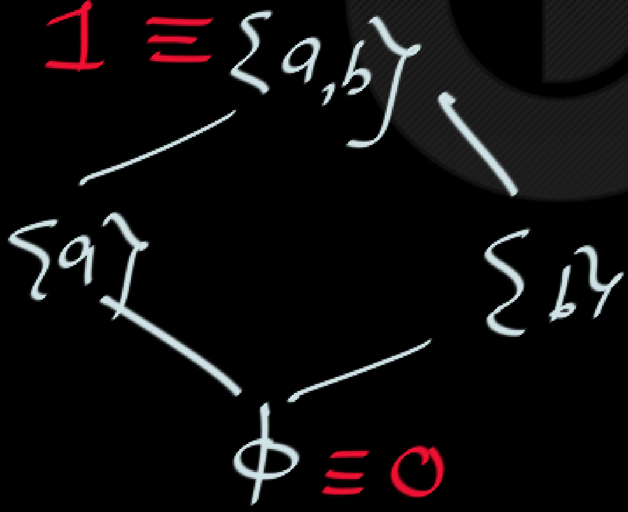
Important & Standard Lattice:

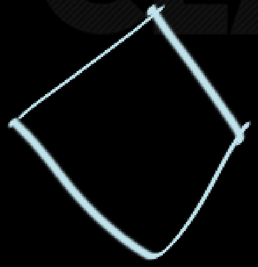
$$P(S), \subseteq$$

$\underbrace{(\mathcal{P}\{a,b\}, \subseteq)}$ $\longrightarrow$ POSET

$\subseteq$ $\longrightarrow$ Subset Relation $\longrightarrow$ POR

$(\{\phi, \{a\}, \{b\}, \{a,b\}\}, \subseteq)$: Boolean Algebra ✓



$\equiv$ 

$\{a\} \vee \{b\} = \{a,b\}$

$\{a\} \wedge \{b\} = \phi$

$\overline{\{a\}} = \{b\}$; $\overline{\phi} = \{a,b\}$

S : Any set

$(P(S), \subseteq)$: → Always POR

(1) POSET ✓

(2) Lattice ✓

(3) Bounded $\begin{cases} \text{Greatest} = S \\ \text{Least} = \phi \end{cases}$

(4) LUB ≡ Union
GLB ≡ Intersection

(5) $\bar{A} = S \setminus A = S - A$

(6) Complemented ✓

(7) Distributive ✓

(8) Boolean Lattice ✓

S : Any set

$(P(S), \subseteq)$

Set theory : $\longrightarrow$ Union, Intersection Distribute over each other.

$$\left\{\begin{array}{l} A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{array}\right\}$$

for any set A, B, C:

$$\left\{\begin{array}{l} A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{array}\right\}$$ In SET chapter.

$\underbrace{(P(S), \subseteq)}$: $GLB \equiv \wedge \equiv$ Intersection $\cap$

$LUB = \vee =$ Union $\cup$

check Distributive property:

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

$$x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$$

S : Any set $|S| = n$

$(P(S), \subseteq)$: Boolean Lattice ✓

2^n elements

$$\left\{ \begin{array}{l} GLB = \text{Intersection} \\ LUB = \text{Union} \\ \overline{A} = S - A \end{array} \right\}$$

NOTE:

For EVREY Set S,

$P(S), \subseteq$ is a Boolean Algebra.

Important Theorem for Boolean Lattice

Theorem:

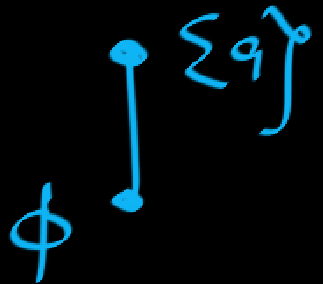
EVERY Boolean Algebra (BA) has same structure as some Powerset Lattice.

$BA \equiv (P(S), \subseteq)$ for some set S.

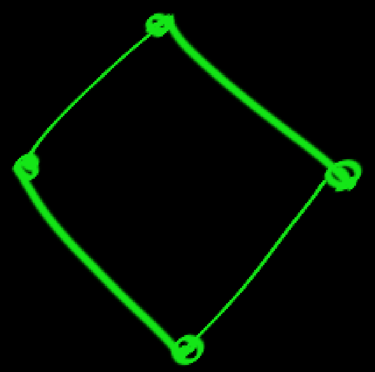
same structure

$(P(S), \subseteq) \longrightarrow BA$

for any S $\longleftarrow$ same structure

S	$(P(S), \subseteq)$	$\equiv$ Boolean Algebra
ϕ $\lvert S \rvert = 0$	$\lvert P(S) \rvert = 2^0 = 1$ ϕ •	• a One element BA
$\{a\}$ $\lvert S \rvert = 1$	$\lvert P(S) \rvert = 2^1 = 2$  	 2 element BA

S	$(P(S), \subseteq)$	Boolean Alg.
$\lvert S \rvert = 2$	$\lvert P(S) \rvert = 2^2 = 4$	
$S = \{a, b\}$		



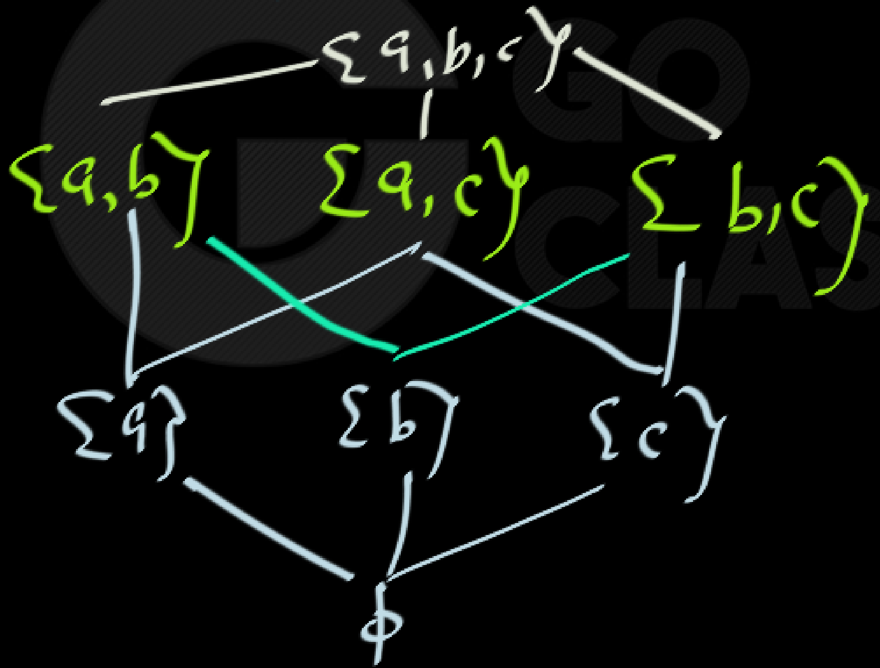
$\underbrace{\text{4 element BA}}$ → unique Structure

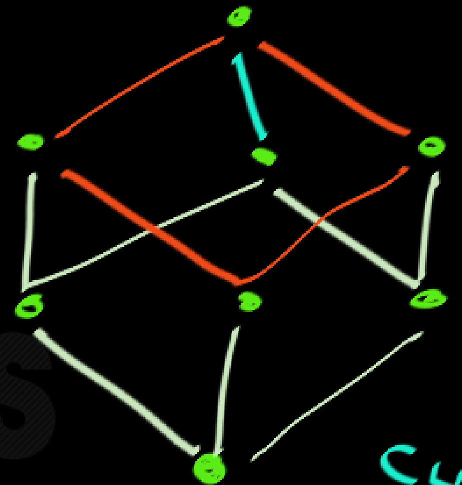
$S \quad | \quad (P(S), \subseteq) \equiv BA$

$|S| = 3$

$S = \{a, b, c\}$

$|P(S)| = 8$





cube

8 Element BA → Unique structure

Note:

Any Boolean Algebra → Same Structure as ~~$P(S), \subseteq$~~ Lattice

2^n elements

Boolean Algebras:

B_1 B_2 B_4 B_8 . . . .

BA $\longrightarrow$ 2^n elements

$\longrightarrow$ same structure as $P(S), \subseteq$

Same structure

Theorem (M. H. Stone, 1936)

Every finite Boolean algebra is isomorphic to the Boolean algebra $\langle \text{pow}(S), \subseteq \rangle$ *of a finite set* S.

Corollary

Every finite Boolean algebra has 2^n *elements for some* $n \in \mathbb{N}$.

Q: True / False?

A lattice with 2^n elements is a Boolean Algebra.

Q: True / False?

A lattice with 2^n elements is a Boolean Algebra.

False

Any BA $\xrightarrow{\quad\quad}$ 2^n elements

$\not\longleftarrow$

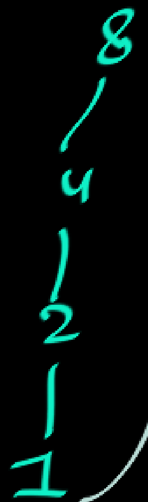
Eg: $(\{1, 2, 4, 8\}, |)$

→ Lattice ✓

→ TOSET ✓

→ 2^2 elements

→ NOT a BA



Not Complemented Lattice

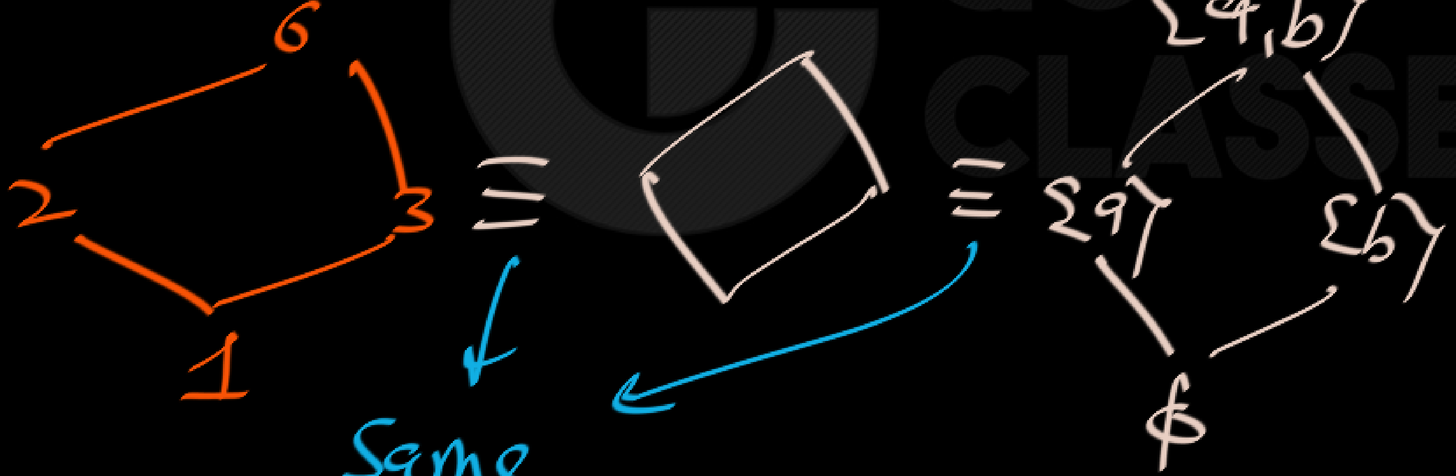
Q: True / False?

A lattice with Not-2^n elements is Not a Boolean Algebra.

Q: ~~True~~ / False?

A lattice with Not-2^n elements is Not a Boolean Algebra. True

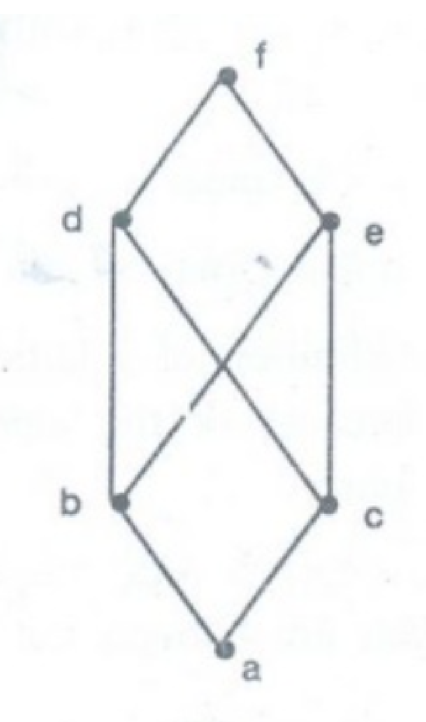
$(\{1, 2, 3, 6\}, |) \to$ BA has same structure as some $(P(S), \subseteq)$ lattice



Same structure

Proposition

For each n, there is a unique Boolean algebra of size 2^n. *There are no other finite Boolean algebras.*



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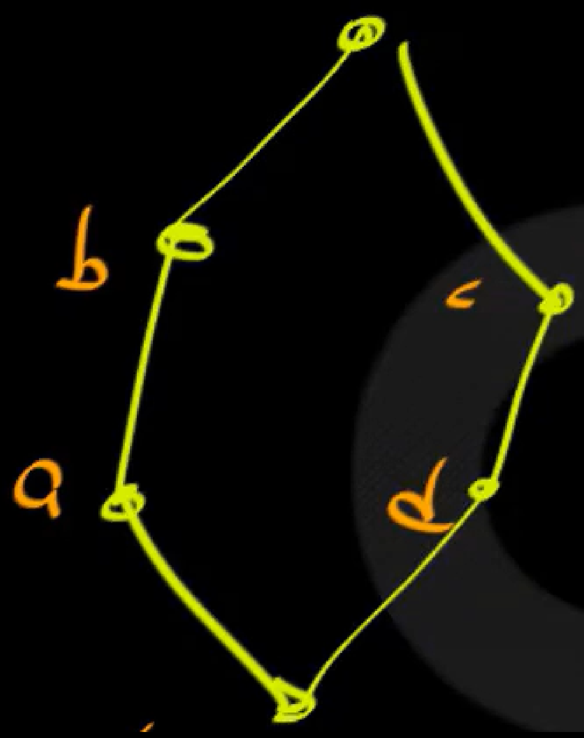
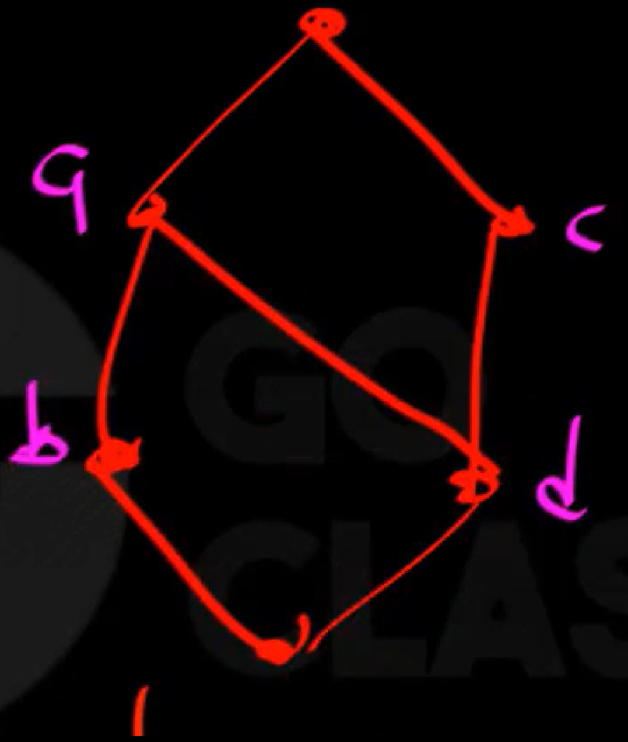




6 elements

6 elements

6 elements

NOT a BA

Complete Analysis of Total Order Relations

Complete Analysis of Total Order Relations

POSET & Every two elements are Comparable.

TOSET :

1. POSET ✓
2. Lattice ✓
3. Distributive Lattice ✓
4. Bounded : may or may not
5. Complemented iff ≤ 2 elements
6. Boolean Algebra iff ≤ 2 elements

$(\mathbb{Z}, \leq)$: TOSET ; <u>NOT Bounded</u>

No Greatest, No least

$(\underbrace{[0,1]}, \leq)$: TOSET ✓ <u>Bounded</u> ✓

0 to 1 Real line interval

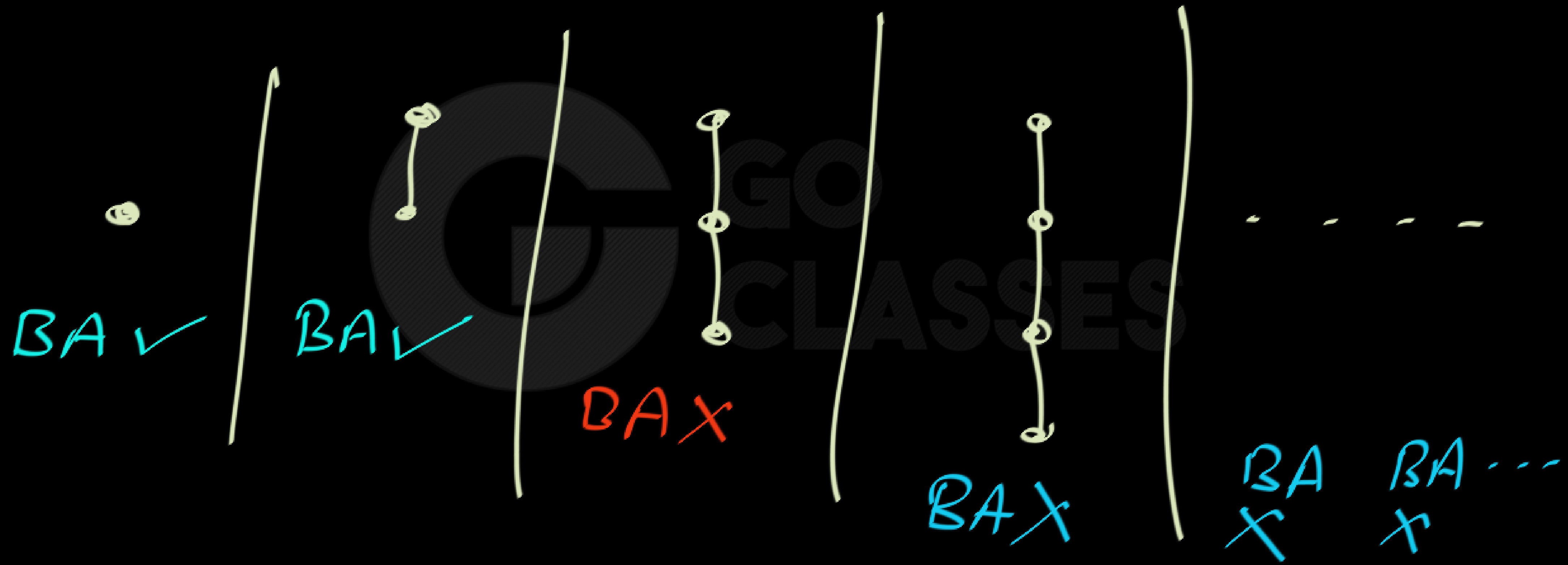
Greatest : 1

Least : 0

$\underbrace{\text{TOSET with } > 2 \text{ elements}}$:
→ **Not Complemented**

$\underbrace{\text{TOSET with } \leq 2 \text{ elements}}$
→ **Boolean Algebra** ✓

TOSET:



Problems 17-20: Totally Ordered Sets and Lattices

Let $\langle \mathcal{A}, \leq \rangle$ be a totally ordered set.

*17. Prove the following.

a. $\mathcal{A}$ is a lattice. → YES

b. What is the meet and join of any two elements a and b in $\mathcal{A}$? Explain.

*18. Prove that if $\mathcal{A}$ has more than two elements, then it is not a complemented lattice, even if it has a minimum and a maximum.

19. Totally ordered sets are ALWAYS distributive lattices.

In TOSET: for any a, b :

$$a \vee b = ?$$

$$a \wedge b = ?$$

In TOSET: for any a, b :

Every two elements are Comparable.

If aRb

$a \vee b = b$

$a \wedge b = a$

If bRa

$a \vee b = a$

$a \wedge b = b$

So, a, b are Comparable

aRb or bRa

In TOSET: for any a, b :

$a \vee b = ? = a$ or b

$a \wedge b = ? = a$ or b

Depending on who is related to whom.

Chain:

b — $a \vee b$

a — $a \wedge b$

Important & Standard Lattice:

$$\underbrace{P(S), \subseteq}$$

Powerset Lattice

$(P(S), \subseteq)$ for any set S

① Lattice ✓

② Boolean Algebra

③ Greatest : S ; Least $= \phi$

④ $GLB = \cap$

$LUB = \cup$

⑤ $\overline{A} = S - A$

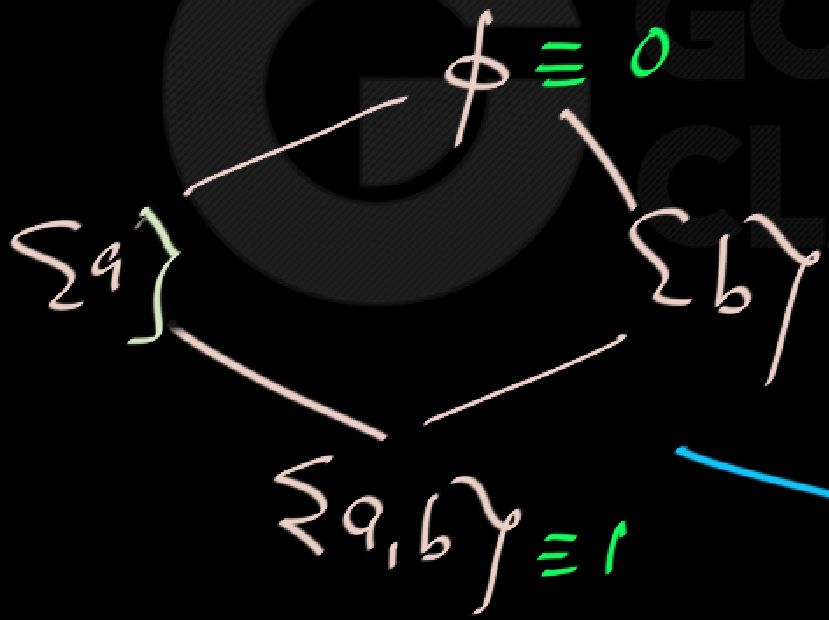
Important & Standard Lattice:

$$P(S), \supseteq$$

$S = \{a, b\};$

$(P(S), \supseteq) \longrightarrow \{a\} \, R \, \phi \quad (\{a\} \supseteq \phi)$

$\phi \, \cancel{R} \, \{a\}$

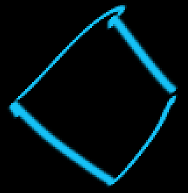


$\{a\} \vee \{b\} =$ Intersection

$\{a\} \wedge \{b\} =$ Union

$\overline{A} = S - A$

$\longrightarrow$ **Boolean Alg**



$(P(S), \supseteq)$ for any set S

$\supseteq$ → Superset

1. Lattice ✓
2. Boolean Algebra ✓
3. Greatest : ϕ ; Least = S
4. LUB ≡ Intersection
   GLB = Union
5. $\overline{A} = S - A$

Important & Standard Lattice:

$D_n, |$

$\underbrace{n \in Z^+}$ $n \longrightarrow 1, 2, 3, 4, \dots$

$D_n \equiv$ Set of all Divisors on n.

Divisors_n

$D_6 = \{1, 2, 3, 6\}$ $6 = 2 \times 3$

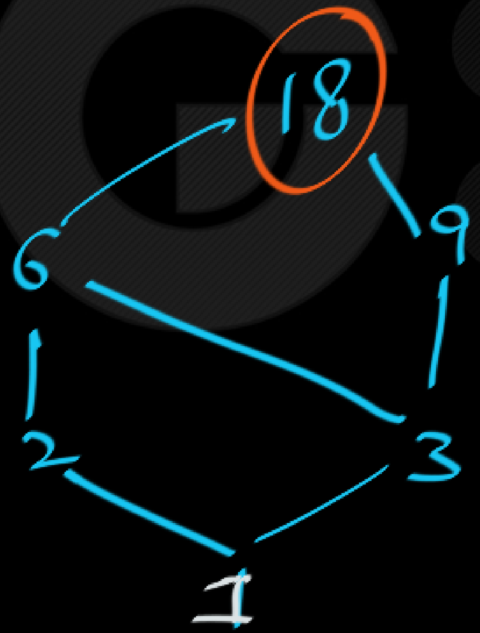
$D_{14} = \{1, 2, 7, 14\}$ $14 = 2 \times 7$

$D_{20} = \{1, 2, 4, 5, 10, 20\}$ $20 = 2^2 \times 5$

$(D_n, |)$ $\quad D_n$: set of ALL Divisors on n.

$D_{18} = \{1, 2, 3, 6, 9, 18\}$ $\quad 18 = 2 \times 3^2$

$(D_{18}, |)$:



BA ✗

Greatest = 18

Least = 1

$6 \wedge 9$

$= GCD$

$= 3$

$2 \vee 3 = 6 = LCM(2,3)$

$6 \vee 9 = LCM(6,9) = 18$

$(D_n, |)$ D_n : Set of ALL Divisors on n.

POSET ✓

Lattice ✓ Bounded ✓

Greatest $= n$; Least $= 1$

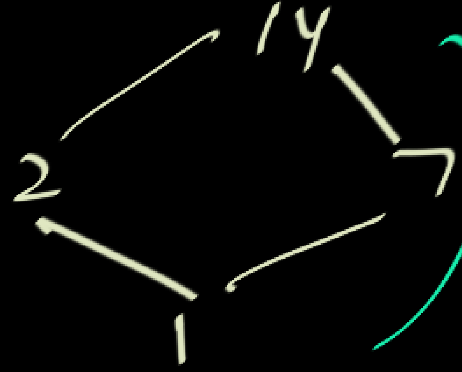
GLB = GCD ; LUB = LCM

Distributive lattice ✓

Is $(D_n, |)$ is Complemented ? → Depends on n.

$(D_{14}, |)$

$D_{14} = \{1, 2, 7, 14\}$



Complemented ✓

$(D_8, |)$

$D_8 = \{1, 2, 4, 8\}$

8
|
4
|
2
|
1

NOT Compl.

$(D_n, 1)$: may or may not be Complemented.

may or may not be BA.

NOTE (Fact from Number Theory):

GCD and LCM Distribute Over Each Other.

Note:

$(D_n, /)$ is Complemented (or BA)

iff n is Squre-free.

Square free : $n \in N$

Prime factorization of $n = \underbrace{p_1^{n_1} p_2^{n_2} p_3^{n_3} \cdots p_m^{n_m}}$

n is Squre free iff every p_i has power 1.

Prime factorization of Square free number $n = p_1^1 p_2^1 p_3^1 \cdots p_m^1$

Square free number's Prime factorization;

$$\underbrace{n = p_1^1 \, p_2^1 \, p_3^1 \cdots p_m^1}$$

No p^2 Divides n, for any prime p.

Note:

$(D_n, |)$ is Complemented

iff

n is Square free (i.e. $n = p_1^1 p_2^1 p_3^1 \cdots p_m^1$)

Note:

$(D_n, |)$ is Complemented

iff

n is Square free (n is NOT Divisible by p^2 for any prime p)

Which is BA?

① $(D_5, |)$

② $(D_6, |)$

③ $(D_7, |)$

④ $(D_8, |)$

⑤ $D_9, |$

⑥ $D_{10}, |$

⑦ $D_{11}, |$

⑧ $D_{12}, |$

Which is BA?

① $(D_5, 1)$ ✓ $5 = 5^1$

② $(D_6, 1)$ ✓ $6 = 2^1 \cdot 3^1$

③ $(D_7, 1)$ ✓ $7 = 7^1$

④ $(D_8, 1)$ ✗ $8 = 2^3$

⑤ $D_9, 1$ ✗ $9 = 3^2$

⑥ $D_{10}, 1$ ✓ $10 = 2^1 5^1$

⑦ $D_{11}, 1$ ✓ $11 = 11^1$

⑧ $D_{12}, 1$ ✗ $12 = 2^2 \cdot 3$

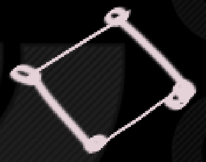
Which is BA?

Another way:

(1) $(D_5, |) \equiv$ 2 elements

BA ✓

(2) $(D_6, |)$ = 4 elements



✓

(3) $(D_7, |)$

(4) $(D_8, |) \Rightarrow$

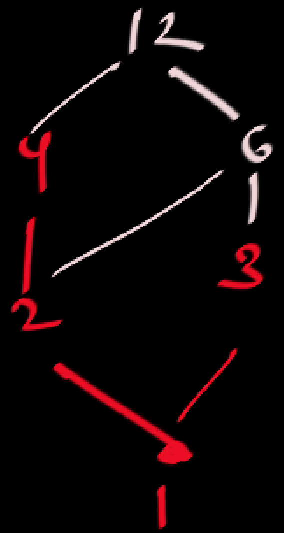
BA ✗

(5) $D_9, |$

(6) $D_{10}, |$

(7) $D_{11}, |$

(8) $D_{12}, |$ → BA ✗



$(D_n, |)$: D_n = Set of ALL Divisors of n.

1. Lattice ✓
2. Bounded Lattice (Greatest = n ; Least = 1)
3. Distributive Lattice ✓
4. LUB = LCM
   GLB = GCD
5. Complemented iff n is square free.
6. BA iff n is square free.

Problems 2-4: Divisor Lattices

The following problems have to do with lattices in which the partial order is the divisibility relation.

*2. Let $\langle \mathcal{D}_{12}, \mid \rangle$ denote the poset of all divisors of 12.

*a. Show that $\mathcal{D}_{12}$ is a lattice by drawing out the Hasse diagram for the poset and then verifying that each pair of divisors has both a meet and a join. How do meet and join relate to the numbers in terms of divisibility?

*b. Is $\mathcal{D}_{12}$ a complemented lattice? Explain.

EC c. Is $\mathcal{D}_{12}$ a distributive lattice? To check whether $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, how many different equations must be checked? Explain.

d. If the bottom point 1 and the top point 12 are deleted from $\mathcal{D}_{12}$, is the result still a lattice? Explain.

e. Is $\mathcal{D}_{12}$ a Boolean lattice? Explain.

Problems 2-4: Divisor Lattices

The following problems have to do with lattices in which the partial order is the divisibility relation.

*2. Let $\langle \mathcal{D}_{12}, \mid \rangle$ denote the poset of all divisors of 12.

*a. Show that $\mathcal{D}_{12}$ is a lattice by drawing out the Hasse diagram for the poset and then verifying that each pair of divisors has both a meet and a join. How do meet and join relate to the numbers in terms of divisibility?

Lcm GCD

*b. Is $\mathcal{D}_{12}$ a complemented lattice? Explain. NO

EC c. Is $\mathcal{D}_{12}$ a distributive lattice? To check whether $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, how many different equations must be checked? Explain.

d. If the bottom point 1 and the top point 12 are deleted from $\mathcal{D}_{12}$, is the result still a lattice? Explain.

e. Is $\mathcal{D}_{12}$ a Boolean lattice? Explain.

$\overline{2}$ = DNE

$\overline{6}$ = DNE

YES (GCD, Lcm Distribute over each other)

Problems 2-4: Divisor Lattices

The following problems have to do with lattices in which the partial order is the divisibility relation.

*2. Let $\langle \mathcal{D}_{12}, \mid \rangle$ denote the poset of all divisors of 12.

*a. Show that $\mathcal{D}_{12}$ is a lattice by drawing out the Hasse diagram for the poset and then verifying that each pair of divisors has both a meet and a join. How do meet and join relate to the numbers in terms of divisibility?

*b. Is $\mathcal{D}_{12}$ a complemented lattice? Explain.

EC c. Is $\mathcal{D}_{12}$ a distributive lattice? To check whether $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, how many different equations must be checked? Explain.

d. If the bottom point 1 and the top point 12 are deleted from $\mathcal{D}_{12}$, is the result still a lattice? Explain.

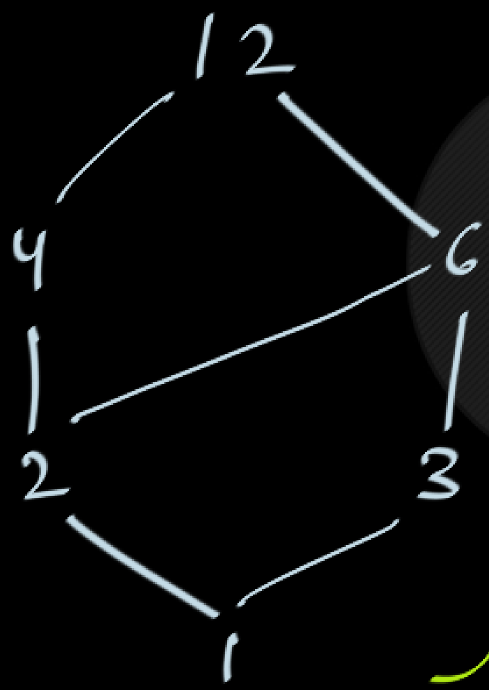
e. Is $\mathcal{D}_{12}$ a Boolean lattice? Explain.

No. (because NOT Complemented)

No.

No; because 12 is NOT square free.

(a) $(D_{12}, |)$ $D_{12} = \{1,2,3,4,6,12\}$ $12 = 2^2 \times 3$



Check Lattice or not :

for Non-Comparable pairs,
Check GLB, LUB exists or NoT.

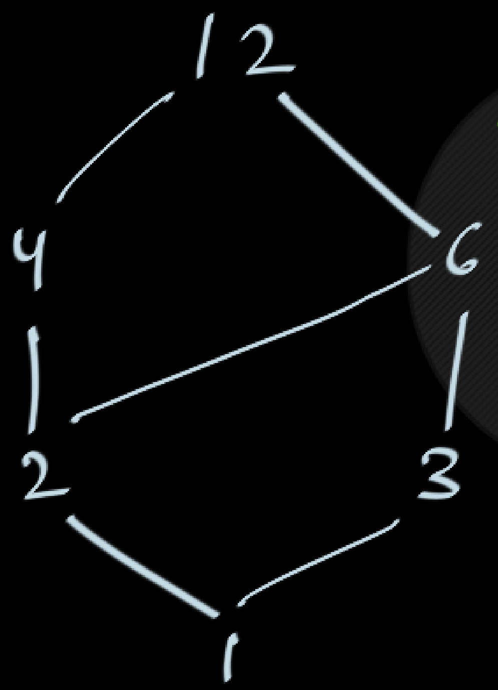
2,3 : $2 \vee 3 = 6$, $2 \wedge 3 = 1$

3,4 : $3 \vee 4 = 12$, $3 \wedge 4 = 1$

4,6 : $4 \vee 6 = 12$, $4 \wedge 6 = 2$

POSET ✓ Lattice ✓

(a) $(D_{12}, |)$ $D_{12} = \{1,2,3,4,6,12\}$ $12 = 2^2 \times 3$



POSET ✓

Lattice ✓

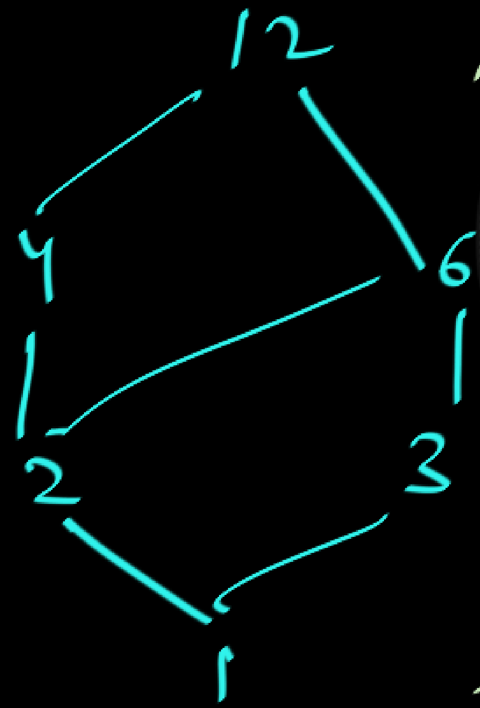
GLB = GCD; LUB = LCM

$4 \wedge 6 = \text{GCD}(4,6) = 2$

$3 \vee 3 = \text{LCM}(3,4) = 12$

$2 \wedge 4 = \text{GCD}(2,4) = 2$

(b) $(D_{12}, |)$: NOT Complemented



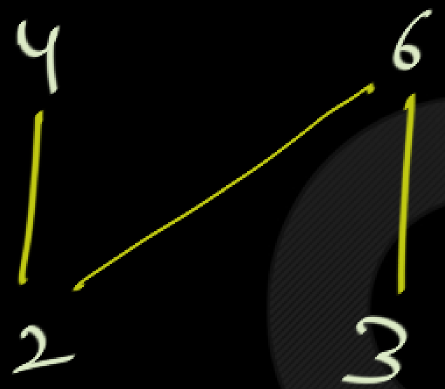
$12 = 2^{2} \times 4$

12 → NOT square free.

$\overline{12} = 1$ $\quad$ $\overline{3} = 4$ $\quad$ $\overline{2} = \text{DNE}$

$\overline{1} = 12$ $\quad$ $\overline{4} = 3$ $\quad$ $\overline{6} = \text{DNE}$

d



NOT a Lattice.

$2 \wedge 3 = \text{DNE}$

$4 \vee 6 = \text{DNE}$

3. Let $\langle \mathcal{D}_{30}, \mid \rangle$ denote the poset of all divisors of 30.
   a. Show that $\mathcal{D}_{30}$ is a lattice. Explain.
   b. Is $\mathcal{D}_{30}$ a complemented lattice? Explain.
   c. Is $\mathcal{D}_{30}$ a distributive lattice? Explain.
   d. Is $\mathcal{D}_{30}$ a Boolean lattice? Explain.

4. Let $\langle \mathcal{D}_n, \mid \rangle$ denote the poset of all divisors of n, where n is a positive integer.
   a. Prove that $\mathcal{D}_n$ is a lattice. Carefully explain why $x \wedge y = \gcd(x, y)$ and $x \vee y = \operatorname{lcm}(x, y)$.
   b. When will $\mathcal{D}_n$ be a complemented lattice? Explain.
   c. When will $\mathcal{D}_n$ be a distributive lattice? Explain.
   d. For which n will $\mathcal{D}_n$ be a Boolean lattice?

3. Let $\langle \mathcal{D}_{30}, \mid \rangle$ denote the poset of all divisors of 30.
   a. Show that $\mathcal{D}_{30}$ is a lattice. Explain.
   b. Is $\mathcal{D}_{30}$ a complemented lattice? Explain.
   c. Is $\mathcal{D}_{30}$ a distributive lattice? Explain.
   d. Is $\mathcal{D}_{30}$ a Boolean lattice? Explain.

$(D_n, |)$: Lattice

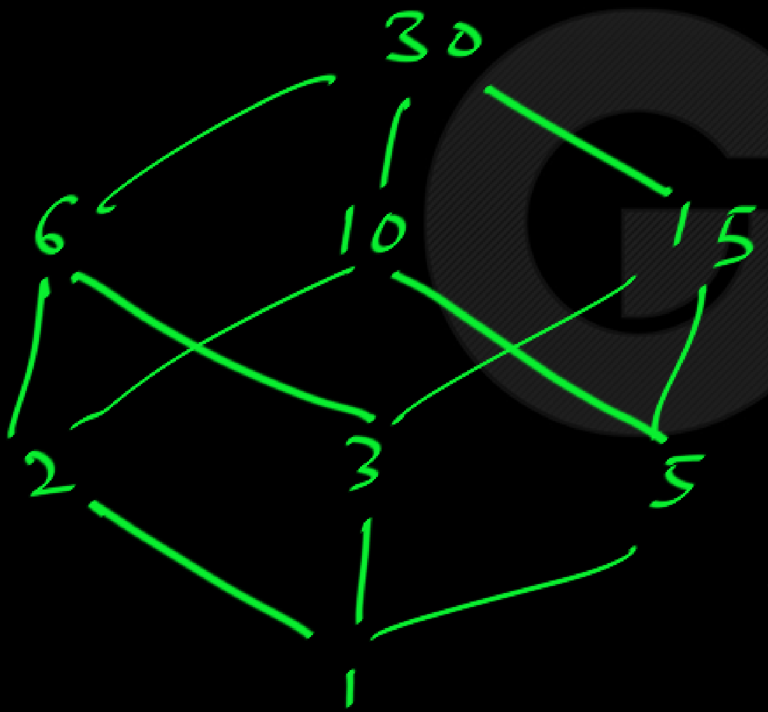
$30 = 2^1 \times 3^1 \times 5^1$

4. Let $\langle \mathcal{D}_n, \mid \rangle$ denote the poset of all divisors of n, where n is a positive integer.
   a. Prove that $\mathcal{D}_n$ is a lattice. Carefully explain why $x \wedge y = \gcd(x, y)$ and $x \vee y = \operatorname{lcm}(x, y)$.
   b. When will $\mathcal{D}_n$ be a complemented lattice? Explain. iff n is square free
   c. When will $\mathcal{D}_n$ be a distributive lattice? Explain.
   d. For which n will $\mathcal{D}_n$ be a Boolean lattice? iff n is square free.

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$30 = \underbrace{2^1 \times 3^1 \times 5^1}$

Prime factorization of 30



Cube $\rightarrow$ a Boolean Algebra

$\equiv (P\{a,b,c\}, \subseteq)$

Problems 5-6: True or False

Are the following statements true or false? Explain your answer.

5. Complements are unique in a complemented lattice.

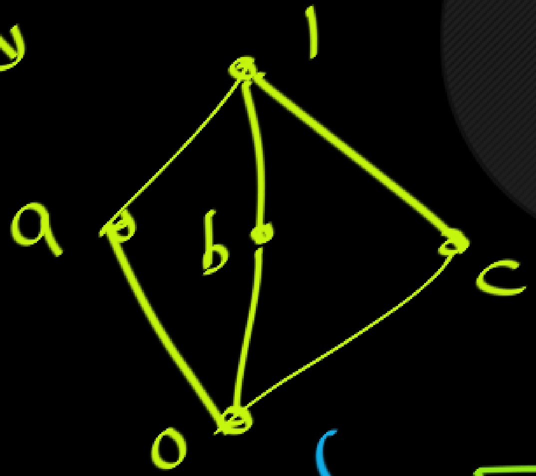
6. If a Boolean lattice has more than two elements, then it is not totally ordered.

Problems 5-6: True or False

Are the following statements true or false? Explain your answer.

5. Complements are unique in a complemented lattice. False

6. If a Boolean lattice has more than two elements, then it is not totally ordered.



$\underbrace{\bar{a} = b, c}$

$\bar{a}$ is NOT unique.

Complemented Lattice

Every element must have at least one complement.

Problems 5-6: True or False

Are the following statements true or false? Explain your answer.

5. Complements are unique in a complemented lattice.

6. If a Boolean lattice has more than two elements, then it is not totally ordered.

True

$\boxed{\text{TOSET is BA iff } n \leq 2}$

12. Suppose $A = \{2, 4, 5, 6, 7, 10, 18, 20, 24, 25\}$ and R is the partial order relation $(x, y) \in R$ iff $x|y$.

(a) Draw the Hasse diagram for the relation.
(b) Find all minimal elements.
(c) Find all maximal elements.
(d) Find all upper bounds for $\{6\}$.
(e) Find all lower bounds for $\{6\}$.
(f) Find the least upper bound for $\{6\}$.
(g) Find the greatest lower bound for $\{6\}$.
(h) Find the least element.
(i) Find the greatest element.
(j) Is this a lattice?

12. Suppose $A = \{2, 4, 5, 6, 7, 10, 18, 20, 24, 25\}$ and R is the partial order relation $(x, y) \in R$ iff $x|y$.

→ Divides

NOT

D_n Lattice

Random numbers

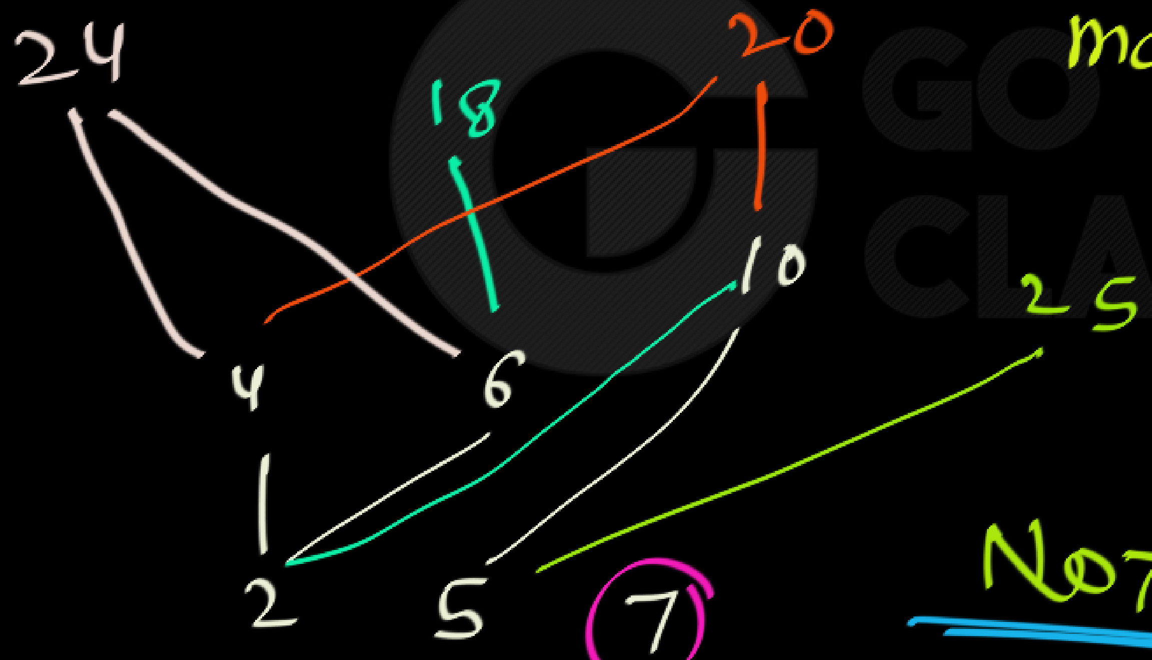
(a) Draw the Hasse diagram for the relation.
(b) Find all minimal elements.
(c) Find all maximal elements.
(d) Find all upper bounds for $\{6\}$.
(e) Find all lower bounds for $\{6\}$.
(f) Find the least upper bound for $\{6\}$.
(g) Find the greatest lower bound for $\{6\}$.
(h) Find the least element.
(i) Find the greatest element.
(j) Is this a lattice?

$\left(A = \{2, 4, 5, 6, 7, 10, 18, 20, 24, 25\}, \mid\right)$

$\boxed{2 \wedge 5 = DNE}$

minimal $= \{2, 5, 7\}$

maximal $= \{24, 18, 20, 25, 7\}$



Greatest : DNE

Least : DNE

NOT a Lattice : $18 \vee 20 = DNE$

Hasse Diagram

$(A = \{2, 4, 5, 6, 7, 10, 18, 20, 24, 25\}, 1)$

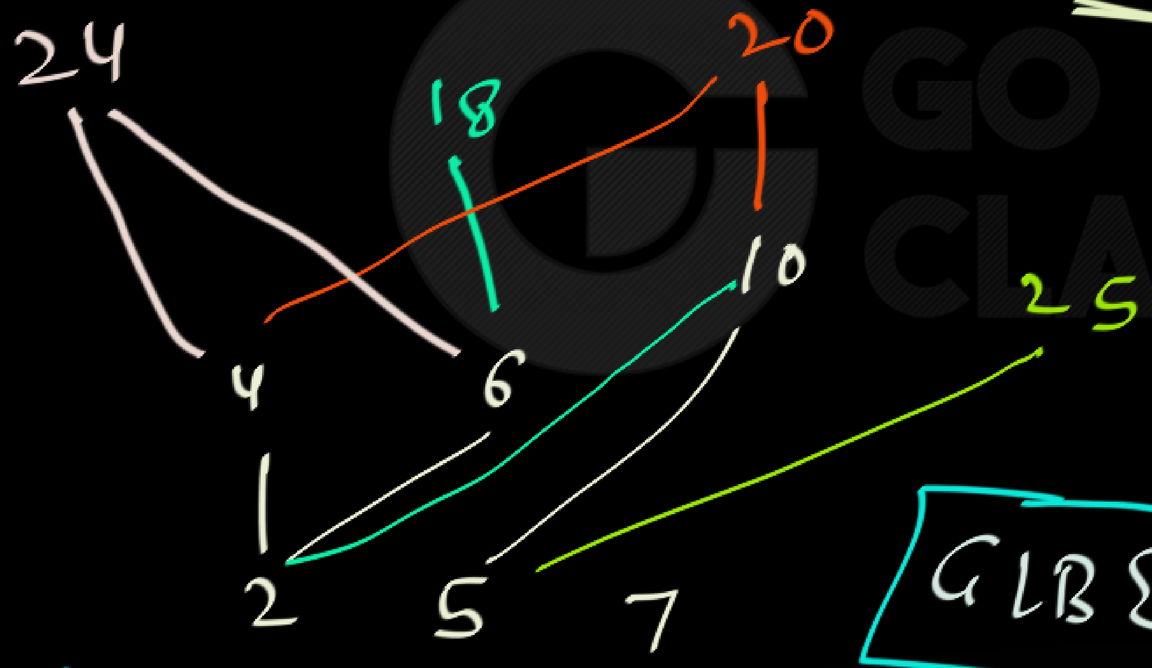
NOT a Lattice.

$UB\{6\} = \{6, 18, 24\}$

$LB\{6\} = \{2, 6\}$

$\boxed{LUB\{6\} = 6 \quad 6 \vee 6 = 6}$

$\boxed{GLB\{6\} = 6 \wedge 6 = 6}$



Hasse Diagram

1. (a) Check with proper justification whether partial order set L in **Figure 1** describes a Lattice or not. (a) Is L distributive or complemented? Justify your answer.

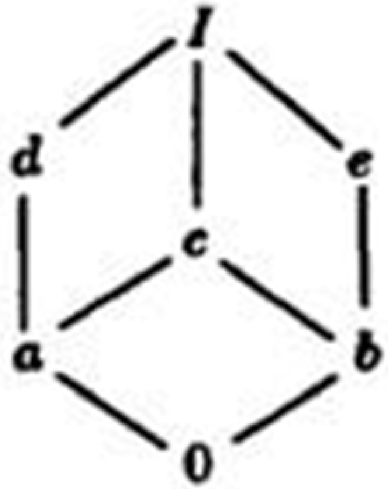
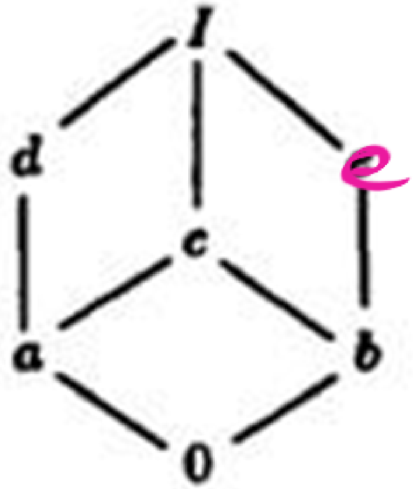


Figure 1

1. (a) Check with proper justification whether partial order set L in **Figure 1** describes a Lattice or not. (a) Is L distributive or complemented? Justify your answer.

Check InComparable pairs for GLB, LUB existence.



Hasse Diagram ↔ POSET

→ Lattice ✓

a, b	a, e	b, d	d, c	d, e	c, e
$a \vee b = c$	$a \vee e = 1$	$b \vee d = 1$	$d \vee c = 1$	$d \vee e = 1$	$c \vee e = 1$
$a \wedge b = 0$	$a \wedge e = 0$	$b \wedge d = 0$	$d \wedge c = a$	$d \wedge e = 0$	$c \wedge e = b$

1. (a) Check with proper justification whether partial order set L in **Figure 1** describes a Lattice or not. (a) Is L distributive or complemented? Justify your answer.

$\bar{c} = b$?? No

$c \vee b = c \neq 1$

c

$\bar{c} = $?

$\bar{c} = d$?? No

$c \wedge d = a \neq 0$

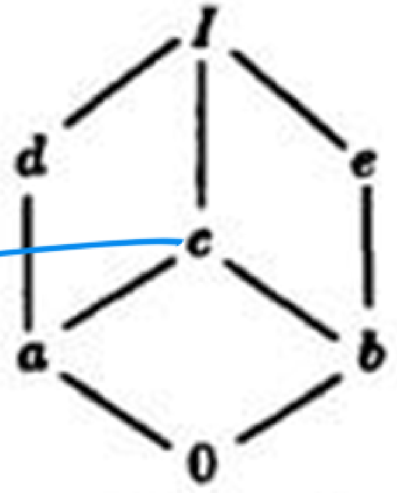


Figure 1

$\bar{c} = \text{DNE}$

NOT Complemented Lattice.

$\bar{d} = e, b \Rightarrow$ NOT Distributive Lattice.

1. (a) Check with proper justification whether partial order set L in **Figure 1** describes a Lattice or not. (a) Is L distributive or complemented? Justify your answer.

Yes. Lattice

Bounded ✓

• Every lattice is Bounded.

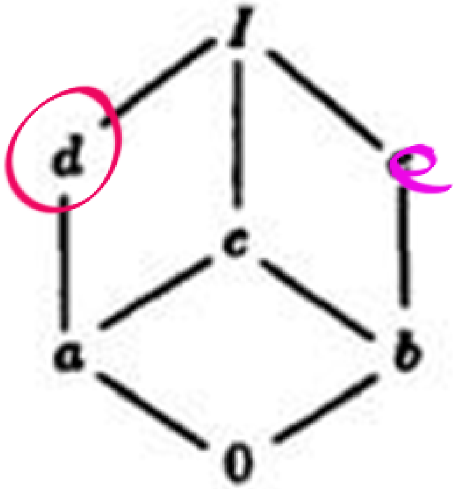


Figure 1

$\bar{d} = e, b$

NOT Distributive

$\bar{e} = d, a$

$\bar{b} = d$; $\bar{a} = e$

$\bar{b} \neq a$

$b \vee a = c \neq 1$

$\bar{0} = 1$; $\bar{1} = 0$; $\bar{c} =$ DNE