

most amazing  
and most needed



Bottom-up parsing  
will get CRYSTAL CLEAR



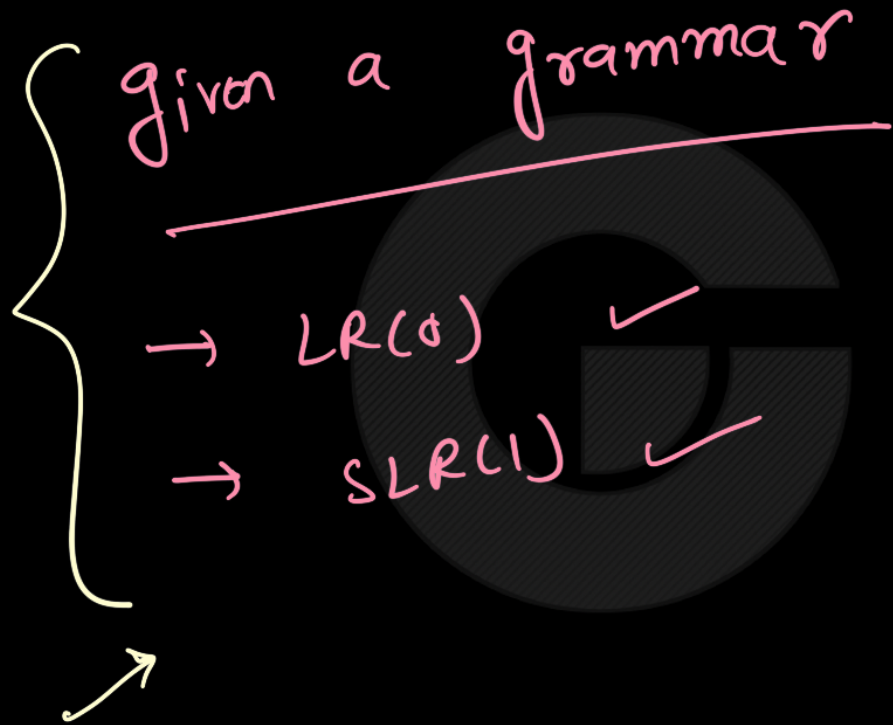
most amazing

Re-Run of DFA

given a grammar

→ LR(0) ✓

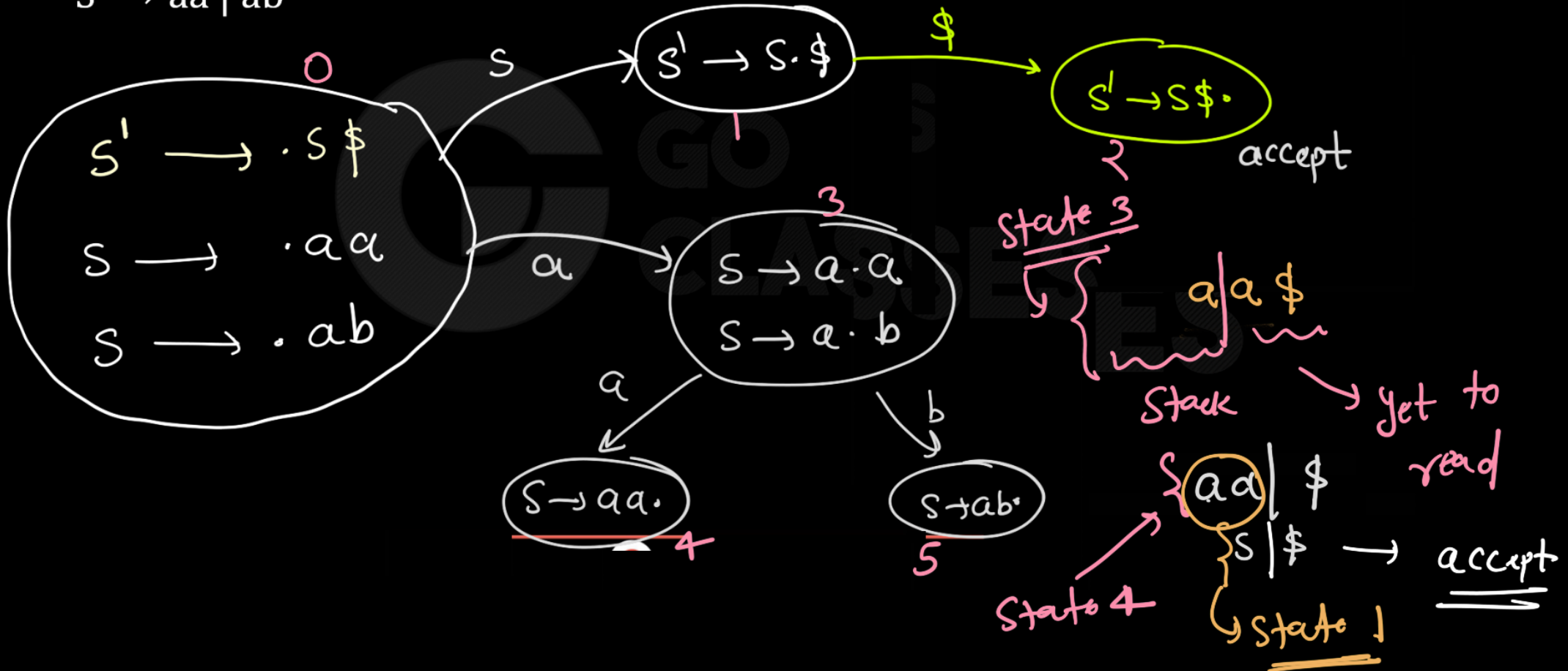
→ SLR(1) ✓

A handwritten diagram in pink and white ink. A pink line underlines the phrase 'given a grammar'. A white curly bracket on the left side groups the two items below: '→ LR(0) ✓' and '→ SLR(1) ✓'. A white arrow points downwards from the bottom of the bracket.

GO  
CLASSES

Build LR(0) automaton

$S \rightarrow aa \mid ab$



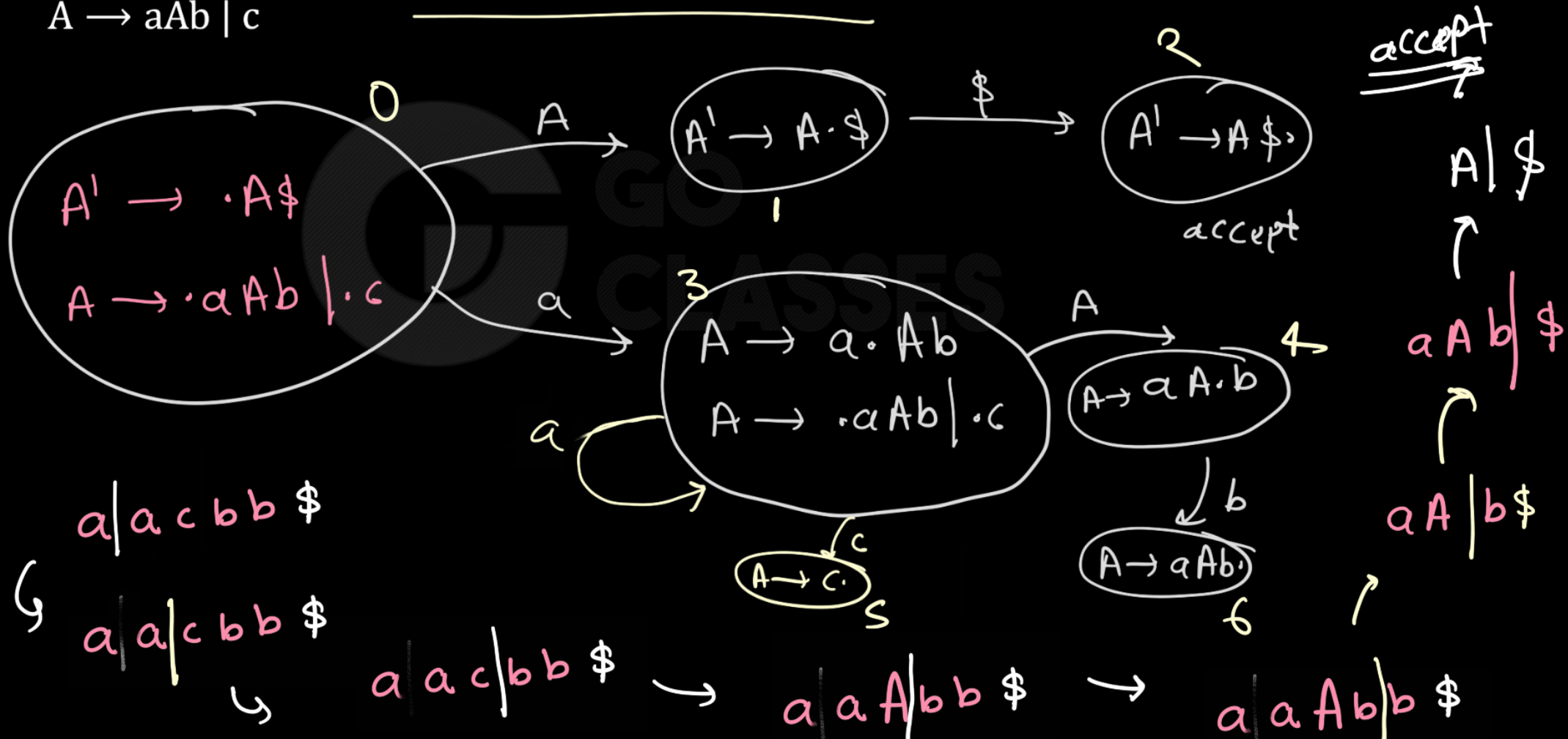
States  
in  
bottomup  
 parsing

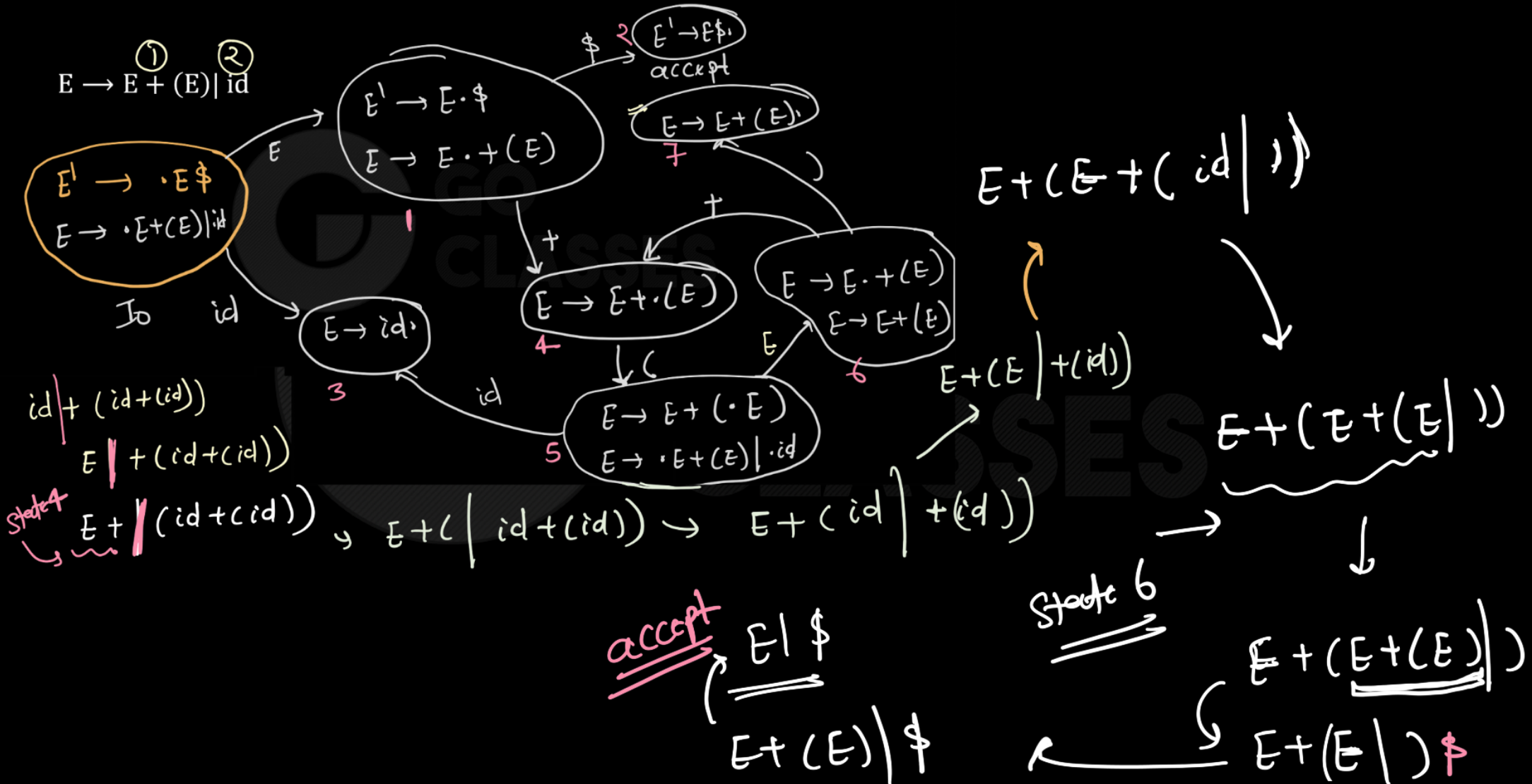


Q: which state the DFA is in?  
 $\Rightarrow$  run DFA on stack input.

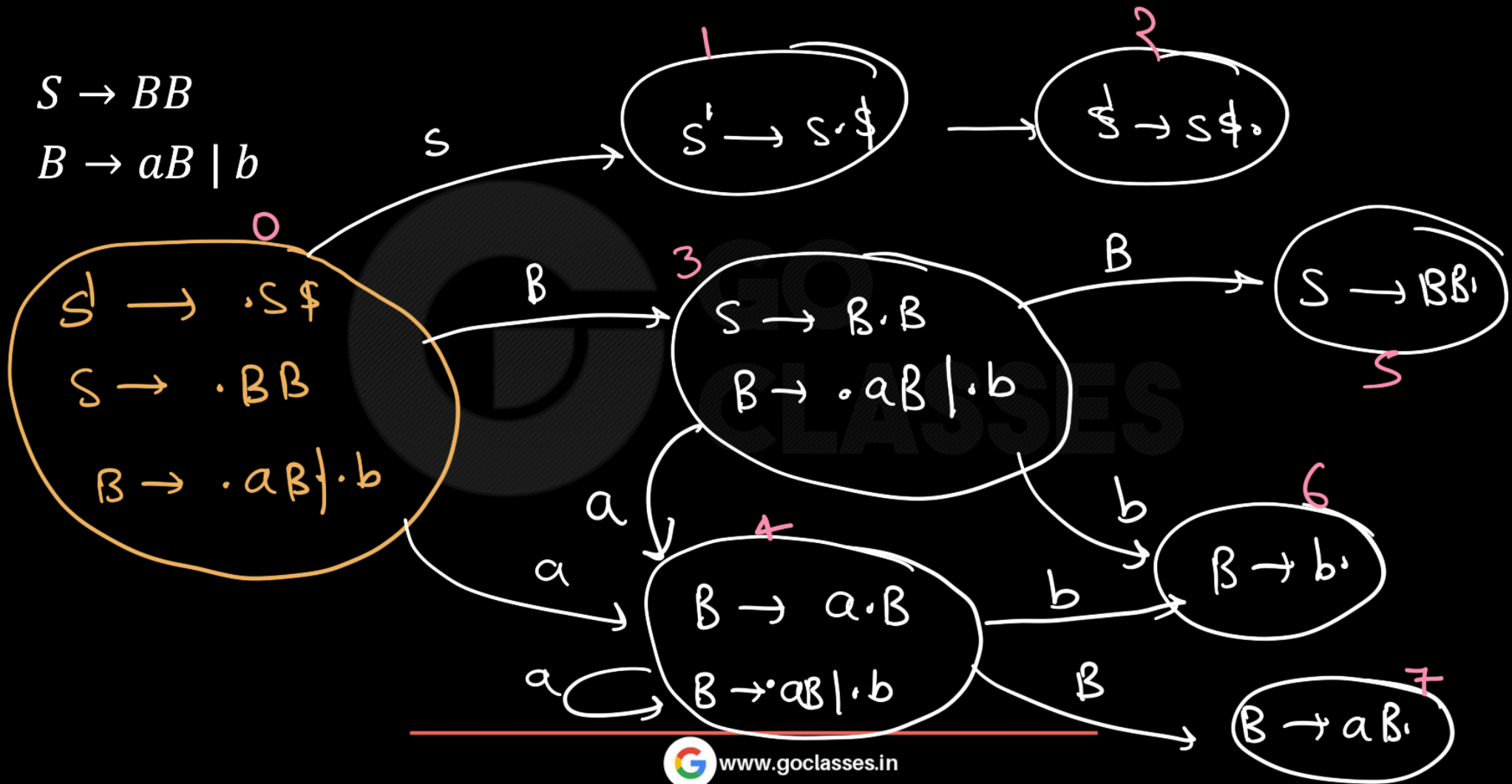
Build LR(0) automaton

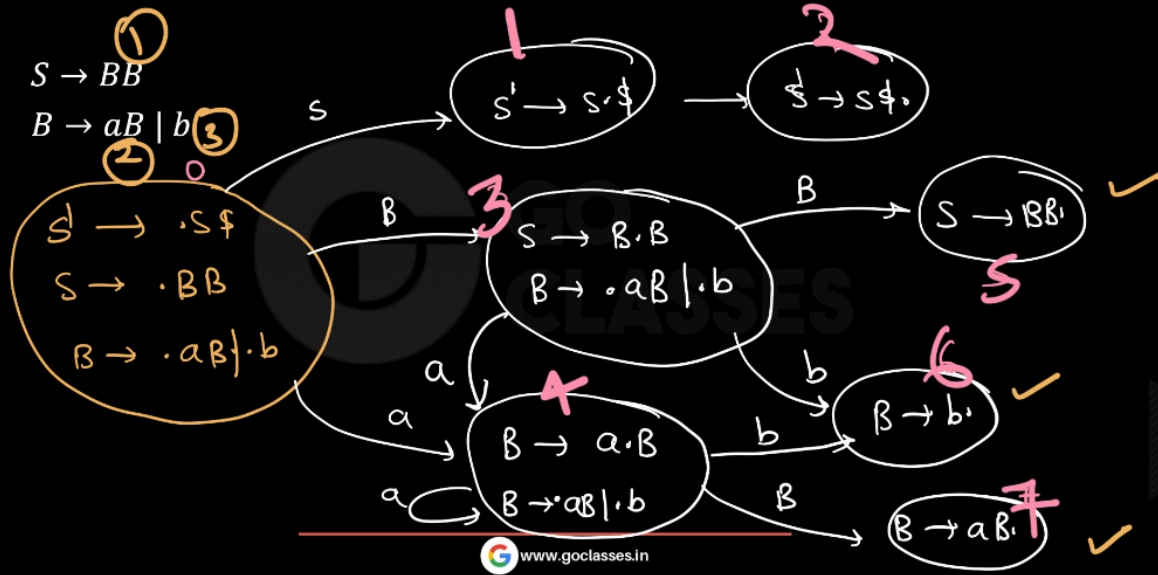
$A \rightarrow aAb \mid c$





$S \rightarrow BB$   
 $B \rightarrow aB \mid b$





Ye-ran DFA

a|a b ab \$

aa|bab \$

aab|ab \$

aaB|ab \$

aB|ab

State 7  
State 3  
B|ab

accept

s|\$

BB|\$

BaB|\$

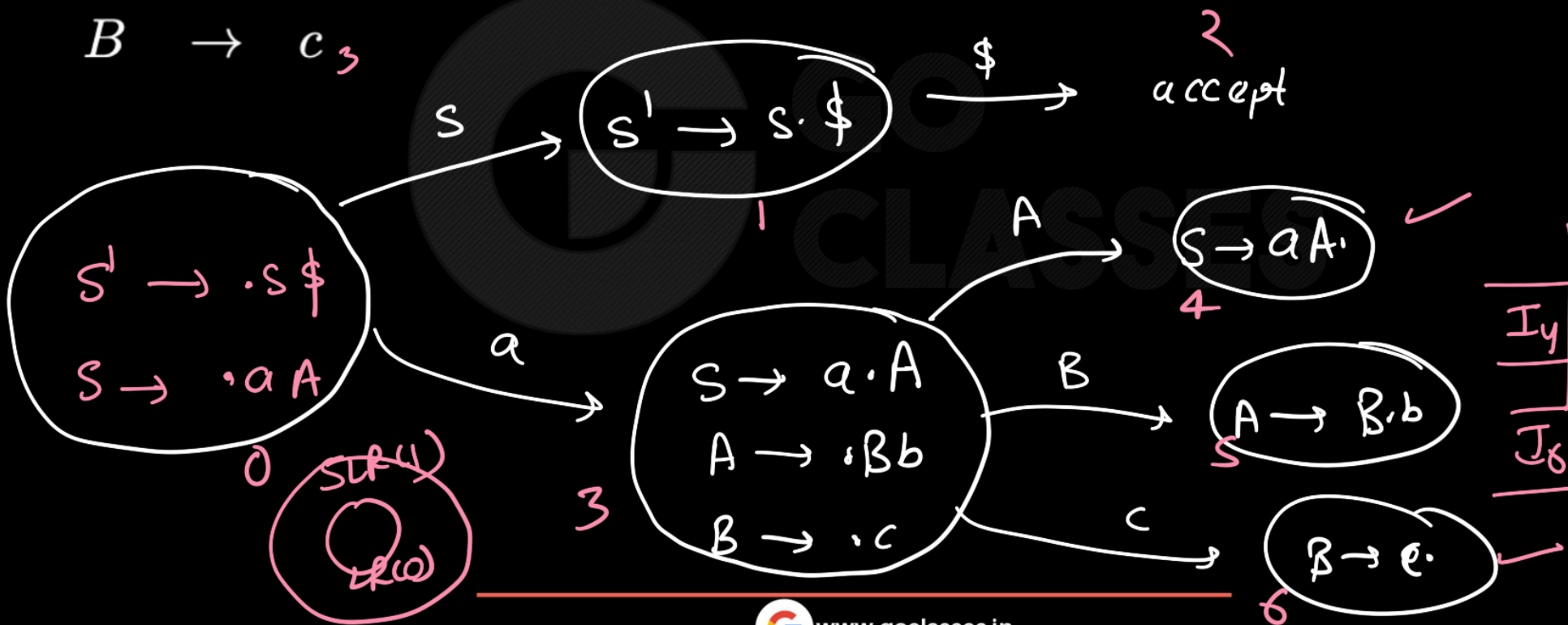
Bab|\$

Ba|b \$

$S \rightarrow aA$  1  
 $A \rightarrow Bb$  2  
 $B \rightarrow c$  3

Is this grammar LR(0)? ✓

Is this grammar SLR(1)? ✓

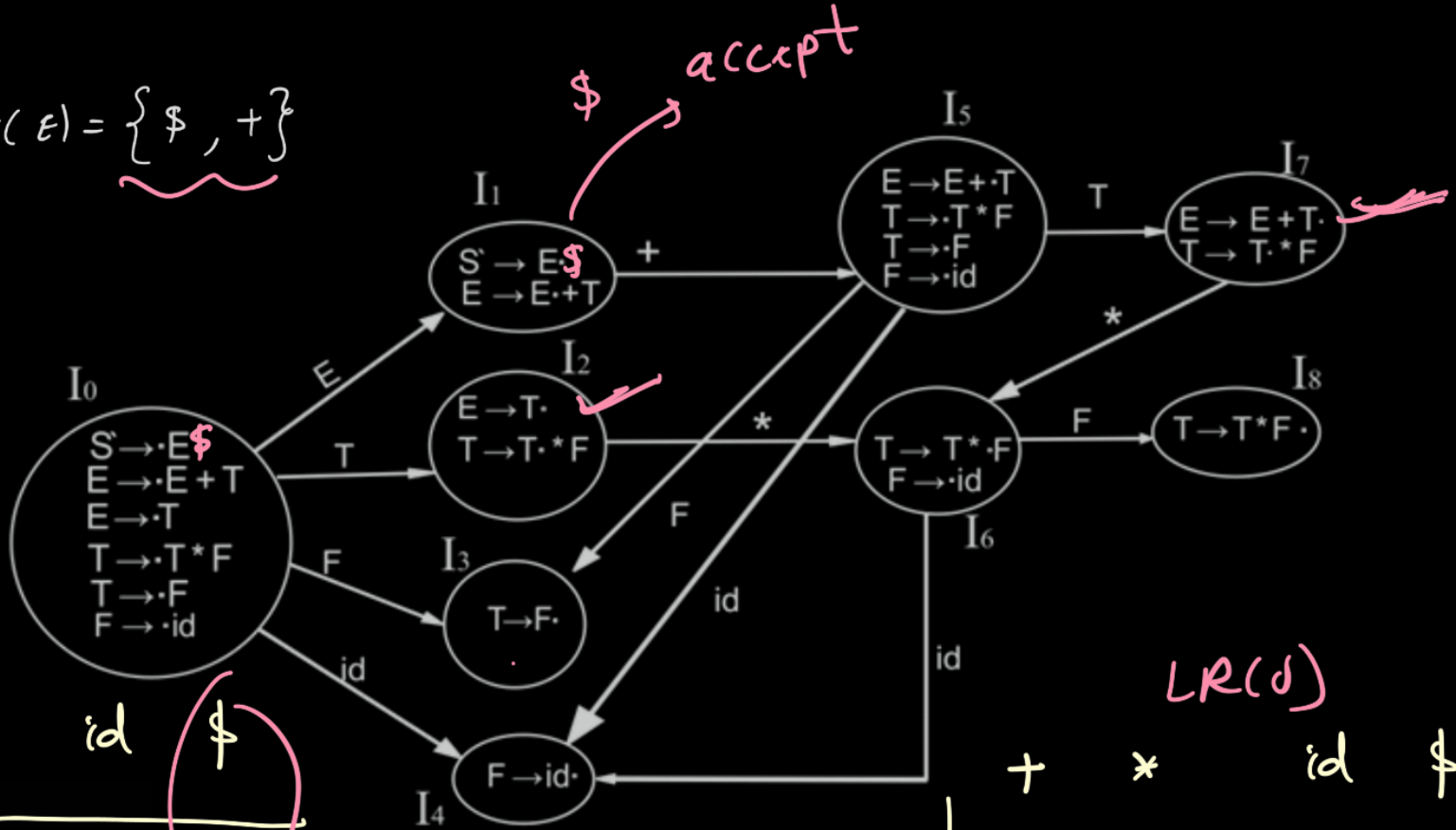


$I_0$	$r_1$	$r_2$	$r_3$
$I_1$	$r_1$	$r_2$	$r_3$

① ②  
 $E \rightarrow E+T \mid T$   
 $T \rightarrow T * F \mid F$   
 $F \rightarrow id$

$follow(E) = \{ \$, + \}$

Is this grammar LR(0)?   
 Is this grammar SLR(1)?



SLR(1)

	+	*	id	\$
$I_2$	$r_2$	$s_6$		$r_2$
$I_7$	$r_1$	$s_6$		$r_1$

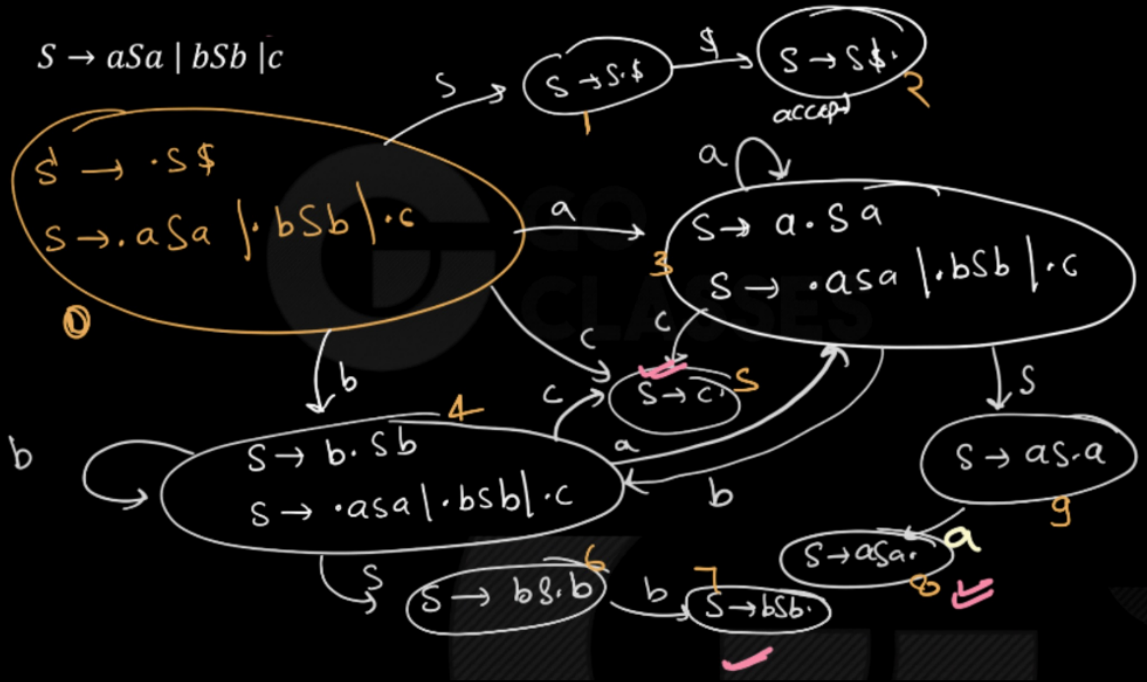
LR(0)

	+	*	id	\$
$I_2$	$r_2$	$r_2/s_6$	$r_2$	$r_2$
$I_7$	$r_1$	$r_1/s_6$	$r_1$	$r_1$

Questions based  
on  
LR(0) Automata states

## MSQ Question

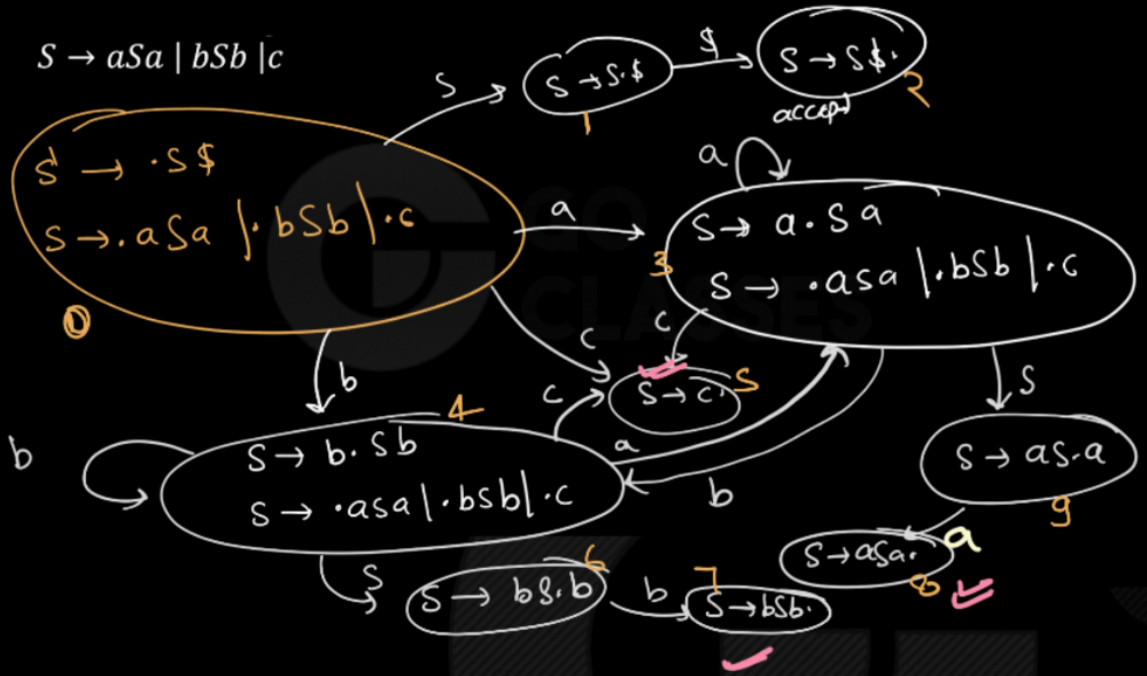
Consider the LR(0) automaton given for the following grammar. Suppose that parser is in state 5 then what are the possible contents on stack ?



- A. The stack content could be "bbbc" where 'c' is on the top of the stack.
- B. The top of the stack must be 'c'.
- C. The stack content could be "bcbc" where 'c' is on the top of the stack.
- D. The stack content could be "bSc" where 'c' is on the top of the stack.

## MSQ Question

Consider the LR(0) automaton given for the following grammar. Suppose that parser is in state 5 then what are the possible contents on stack ?



- A. The stack content could be "bbbc" where 'c' is on the top of the stack.
- B. The top of the stack must be 'c'.
- C. The stack content could be "bcbc" where 'c' is on the top of the stack.
- D. The stack content could be "bSc" where 'c' is on the top of the stack.

$X \rightarrow d \cdot \beta$

what can you say  
about this state?

$\alpha \rightarrow \alpha \cdot \beta$

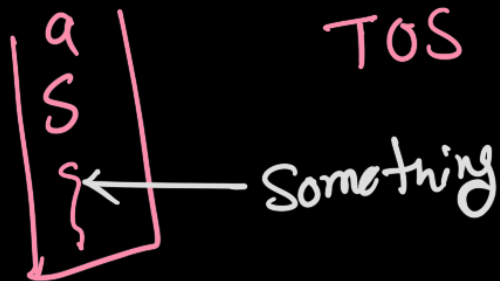
$\Rightarrow \alpha$  must be on TOS

$S \rightarrow Sa.b$   
 $B \rightarrow a.$

← what is on  
TOS?  
==

TOS is "a"

TOS is "Sa"

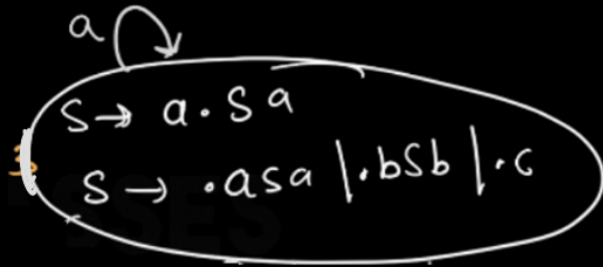




## MSQ Question

Consider the LR(0) automaton state of some unknown grammar.

If parser is in given state then what can we say about content of the stack ?



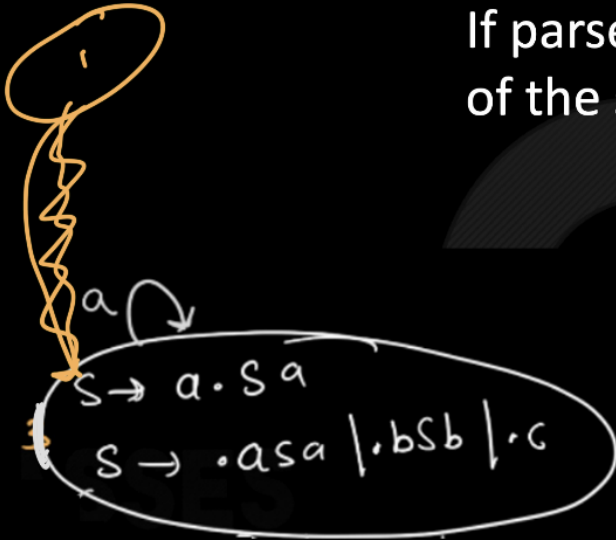
- A. We can not say anything about top of stack
- B. The top of the stack must be 'a'.
- C. The top of the stack must be 'b'.
- D. The top of the stack must be 'c'.



## MSQ Question

Consider the LR(0) automaton state of some unknown grammar.

If parser is in given state then what can we say about content of the stack ?



A. We can not say anything about top of stack

B. The top of the stack must be 'a'.

C. The top of the stack must be 'b'.

D. The top of the stack must be 'c'.

E. stack contain only 'a'



## MSQ Question

Consider the three LR(0) items given of some unknown grammar.

What can we say about these LR(0) items ?

1.  $S \rightarrow a.A$
  2.  $A \rightarrow B.b$
  3.  $B \rightarrow c.$
- A. Item 1 and Item 3 could be part of same LR(0) automaton state.
  - B. Item 1 and Item 2 could be part of same LR(0) automaton state.
  - C. Item 2 and Item 3 could be part of same LR(0) automaton state.
  - D. These 3 items must belong to different-different LR(0) automaton states.

## MSQ Question

Consider the three LR(0) items given of some unknown grammar.

What can we say about these LR(0) items ?

1.  $S \rightarrow \dot{a}.A$
2.  $A \rightarrow B.\dot{b}$
3.  $B \rightarrow \dot{c}$ .

A. Item 1 and Item 3 could be part of same LR(0) automaton state.

B. Item 1 and Item 2 could be part of same LR(0) automaton state.

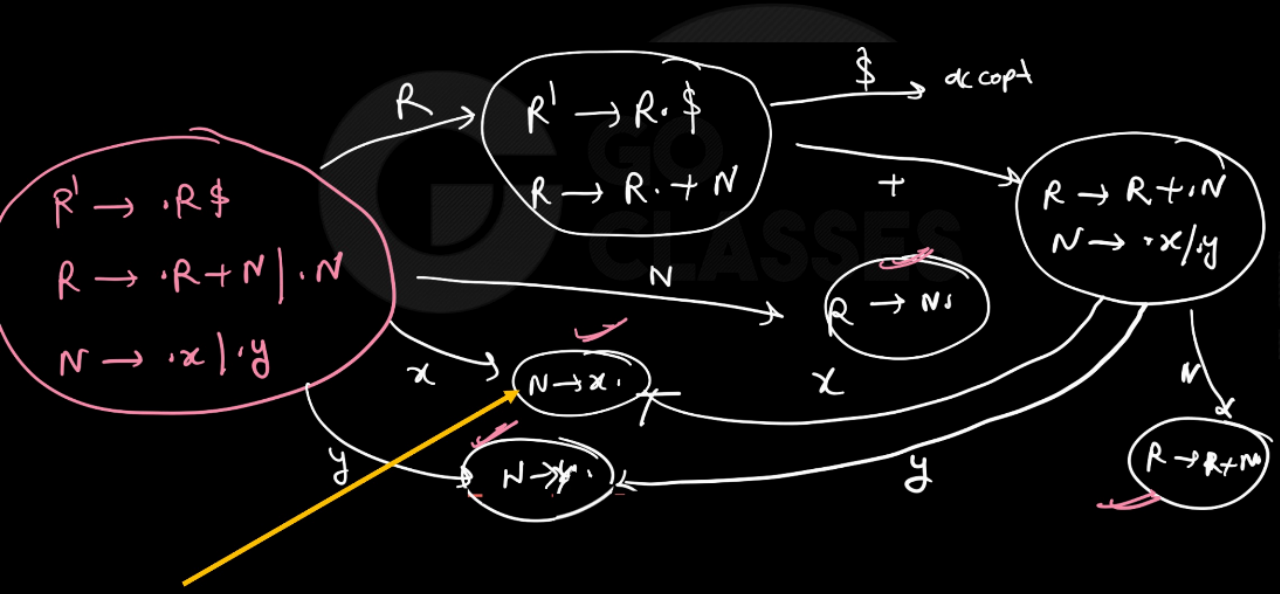
C. Item 2 and Item 3 could be part of same LR(0) automaton state.

D. These 3 items must belong to different-different LR(0) automaton states.

*True*

## MSQ Question

Consider the LR(0) automaton given for the following grammar. Suppose that parser is in the given current state then what are the possible contents on stack ?

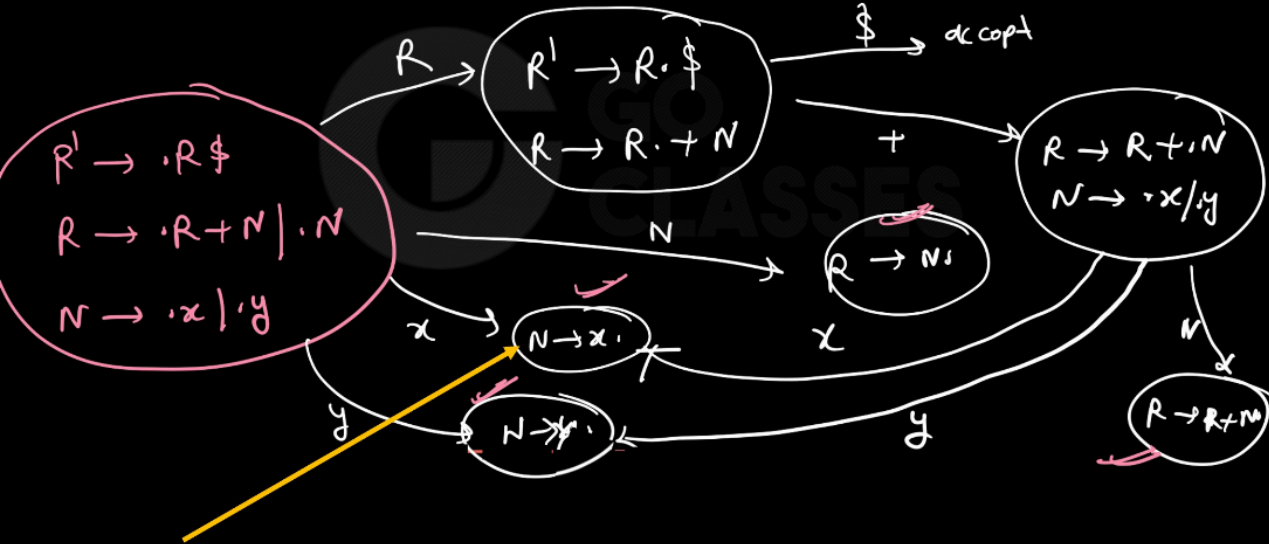


Current state

- A. The stack content could be "R+x" where 'x' is on the top of the stack.
- B. The stack content could be "R+x+R+x" where 'x' is on the top of the stack.
- C. The stack content could be "x"
- D. The top of the stack must be "x"

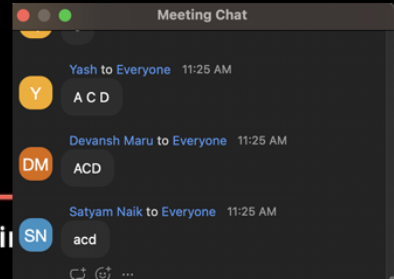
## MSQ Question

Consider the LR(0) automaton given for the following grammar. Suppose that parser is in the given current state then what are the possible contents on stack ?



Current state

- A. The stack content could be "R+x" where 'x' is on the top of the stack.
- B. The stack content could be "R+x+R+x" where 'x' is on the top of the stack.
- C. The stack content could be "x"
- D. The top of the stack must be "x"

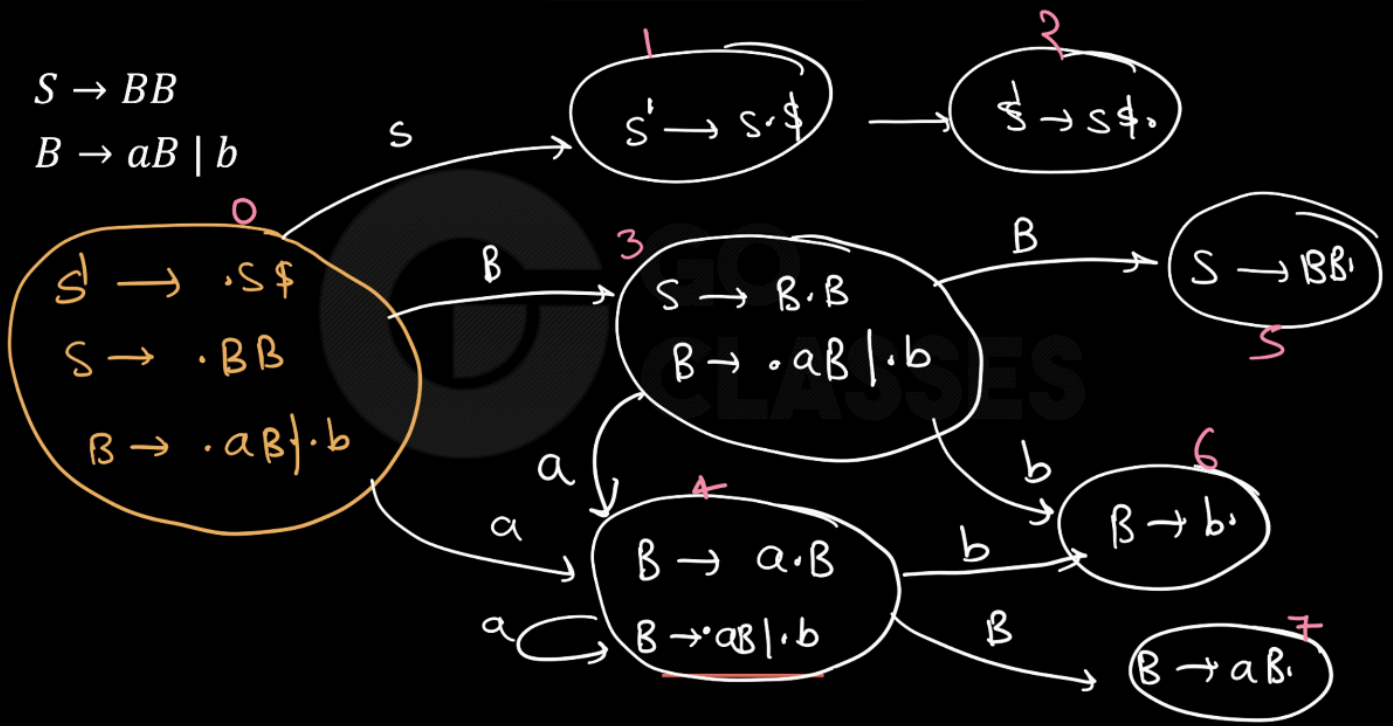


## MSQ Question

Consider the LR(0) automaton given for the following grammar.  
Which of the followings can NEVER be content of the stack in parsing any string ?

$$S \rightarrow BB$$

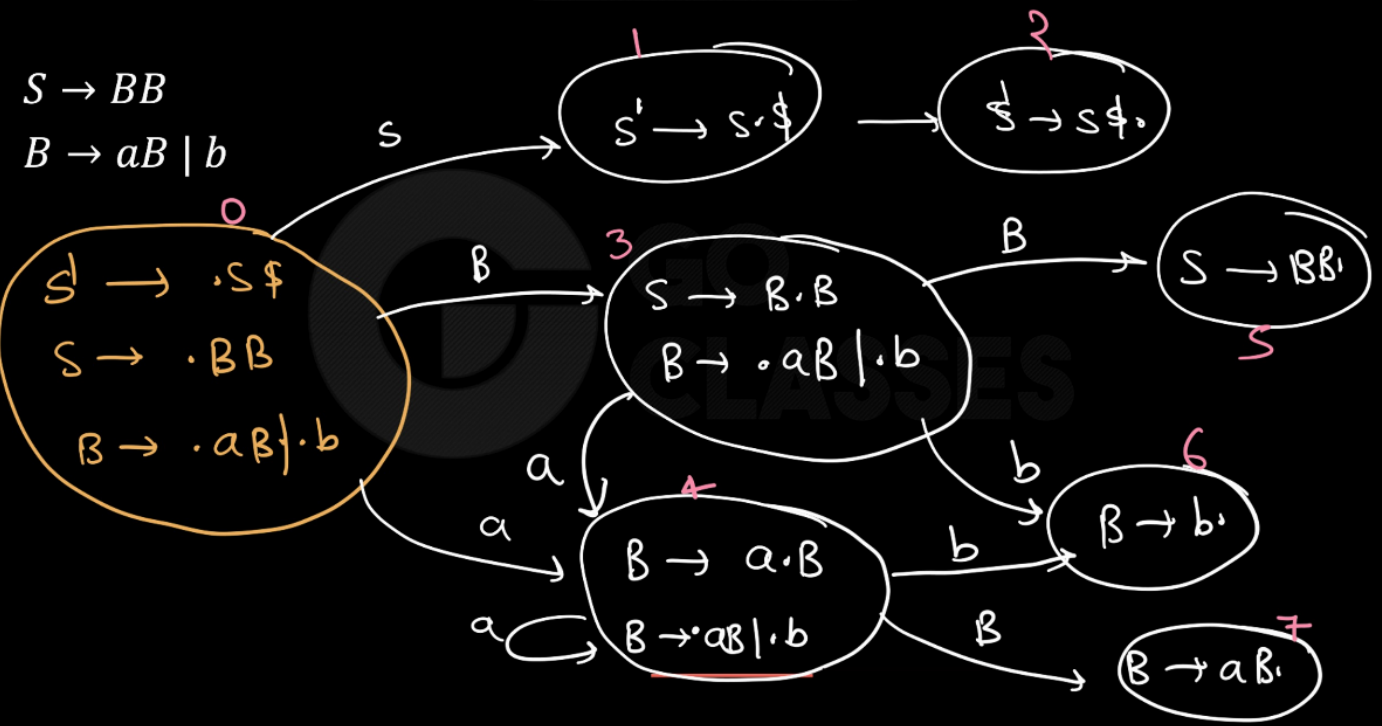
$$B \rightarrow aB \mid b$$



- a) BB
- b) BBb
- c) Bab
- d) aB

## MSQ Question

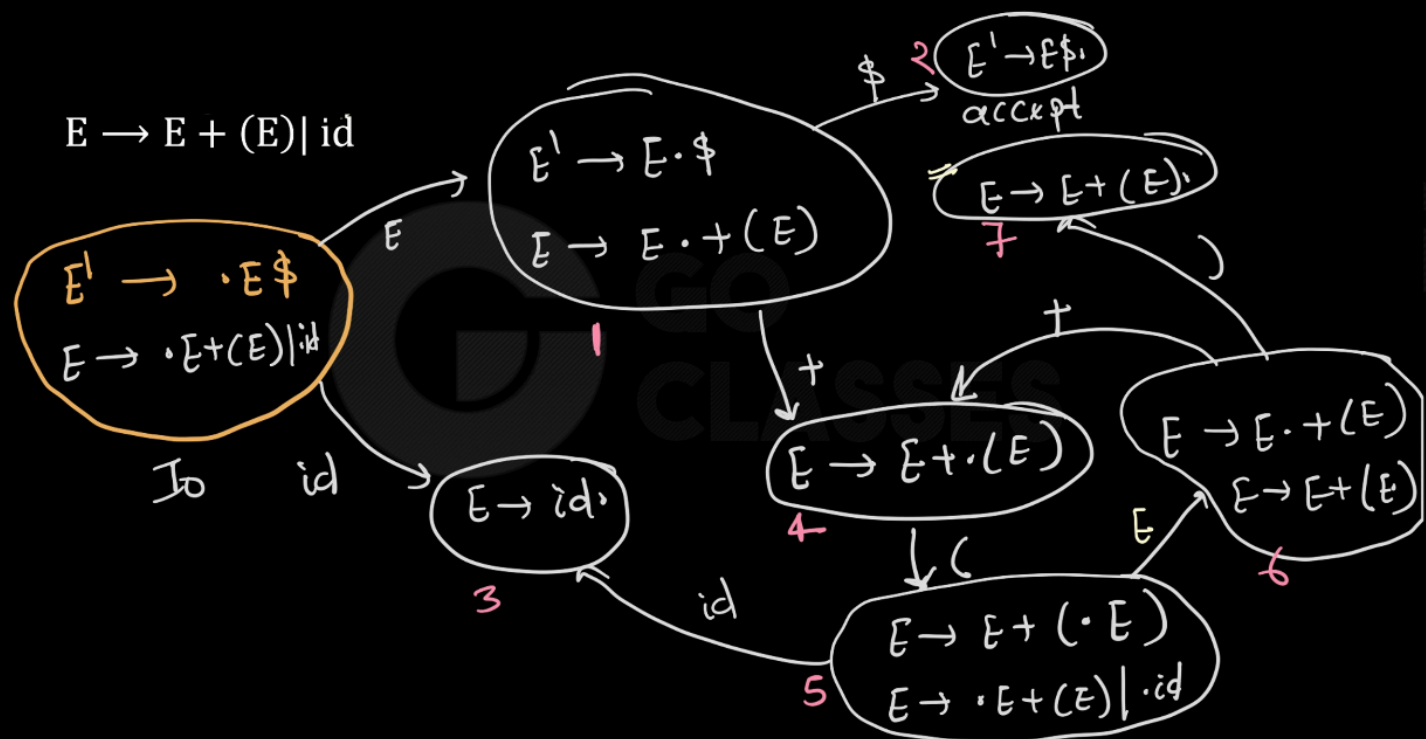
Consider the LR(0) automaton given for the following grammar.  
Which of the followings can NEVER be content of the stack in parsing any string ?



- a) BB
- ~~b) BBb~~
- c) Bab
- d) aB

## MSQ Question

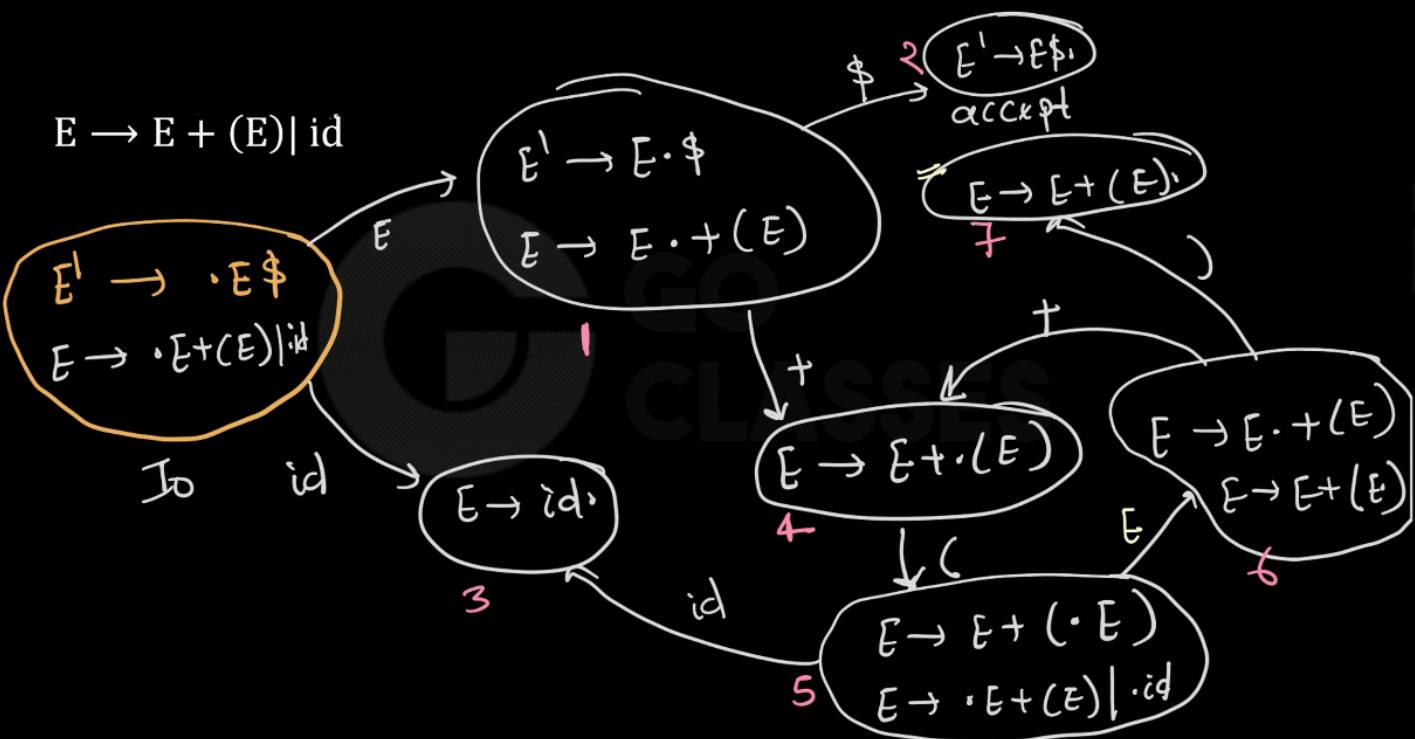
Consider the LR(0) automaton given for the following grammar.  
If parser is state 4 then what can we say about possible contents on the stack ?



1. Stack contain could be "E+" where + is on top of stack
2. Stack contain could be "id+" where + is on top of stack
3. Stack contain could be E+(E+ where + is on top of stack
4. Stack contain could be id+(id+ where + is on top of stack

## MSQ Question

Consider the LR(0) automaton given for the following grammar.  
If parser is state 4 then what can we say about possible contents on the stack ?

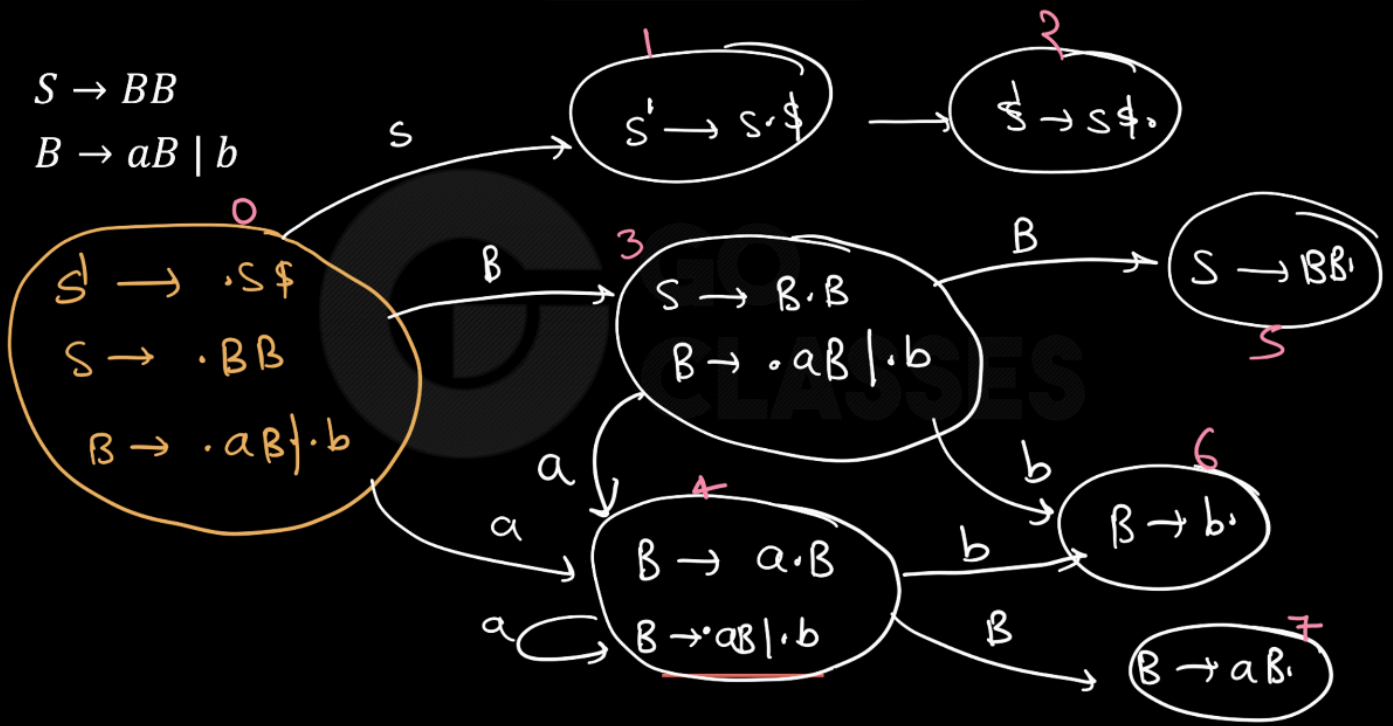


- ✓ Stack contain could be "E+" where + is on top of stack
- ✗ Stack contain could be "id+" where + is on top of stack
- ✓ Stack contain could be E+(E+ where + is on top of stack
- ✗ Stack contain could be id+(id+ where + is on top of stack

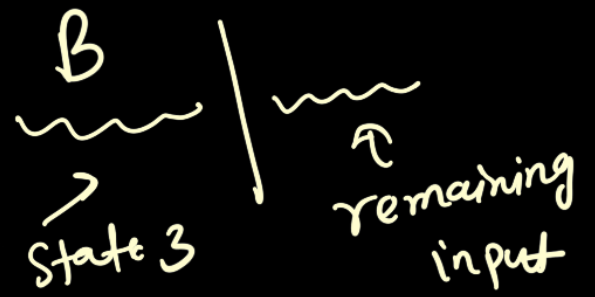
## MSQ Question

Consider the LR(0) automaton given for the following grammar. If parser is state 3 after reading a part of input then what should be remaining input such that input string finally get accepted ?

$S \rightarrow BB$   
 $B \rightarrow aB \mid b$



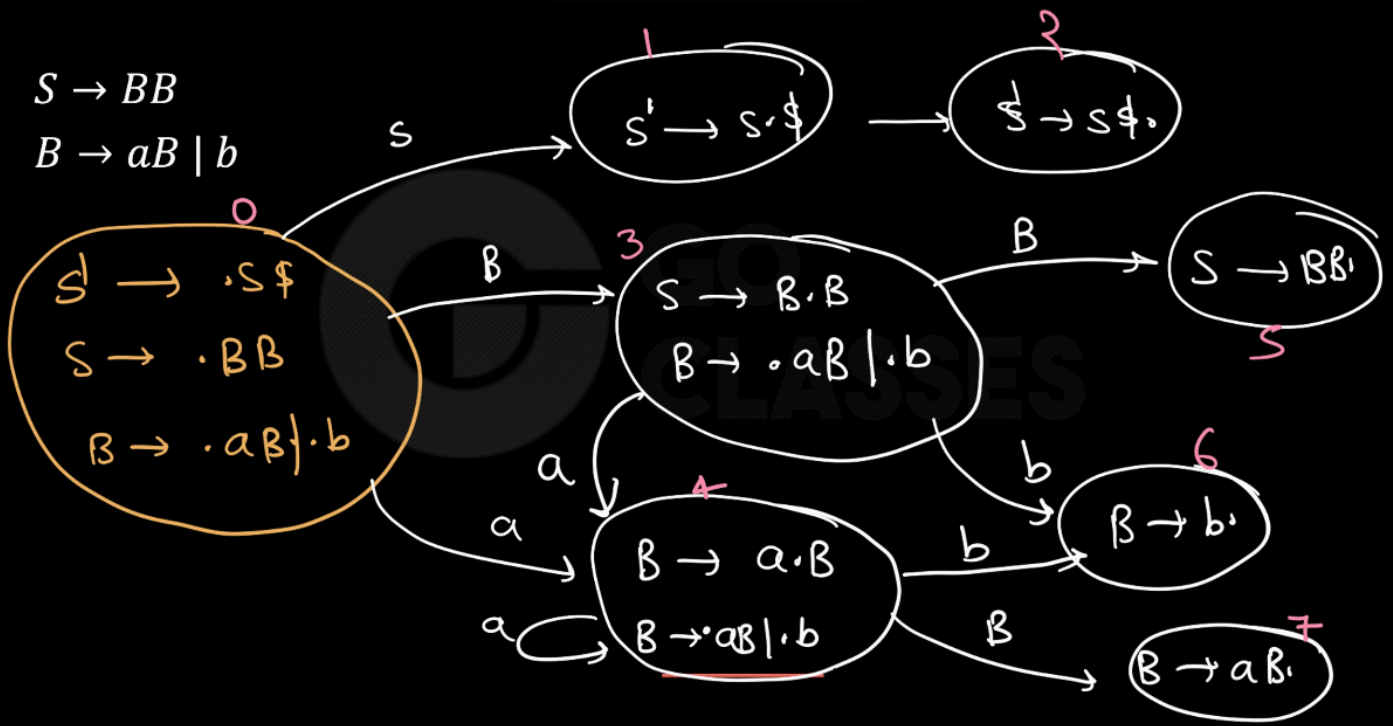
- a) b
- b) ab
- c) abb
- d) aab



## MSQ Question

Consider the LR(0) automaton given for the following grammar. If parser is state 3 after reading a part of input then what should be remaining input such that input string finally get accepted ?

$S \rightarrow BB$   
 $B \rightarrow aB \mid b$

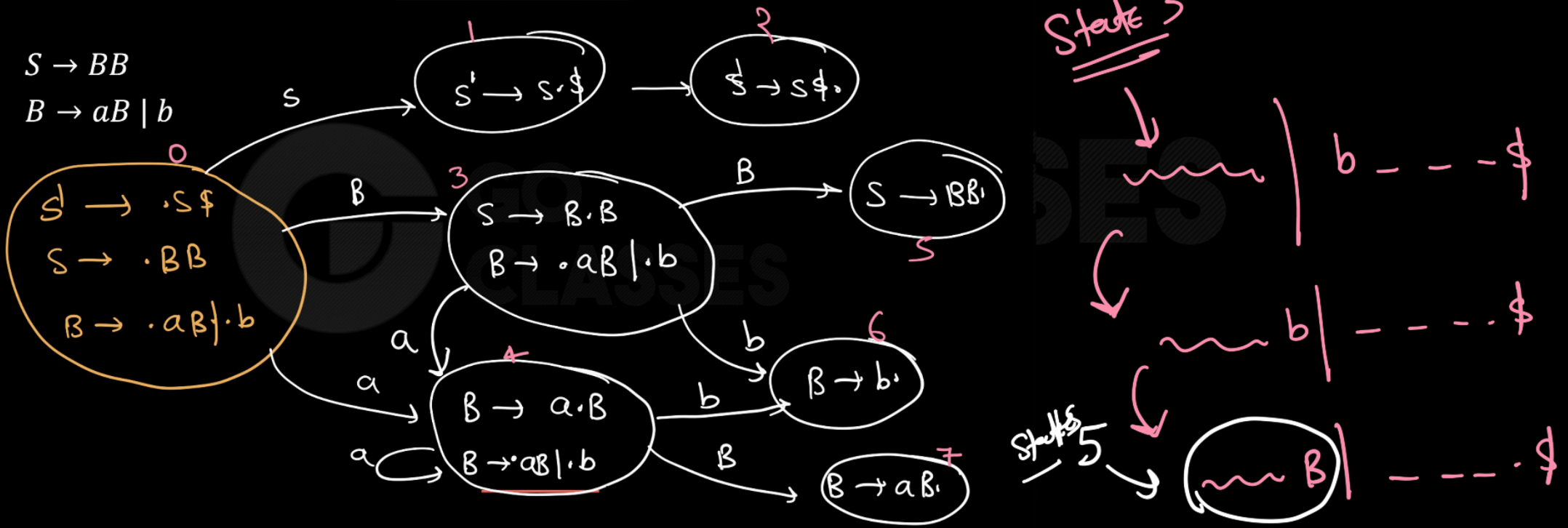


- ~~a) b~~
- ~~b) ab~~
- c) abb
- ~~d) aab~~

~~~~~  
 → State 3  
 ~~~~~  
 ↑  
 remaining input

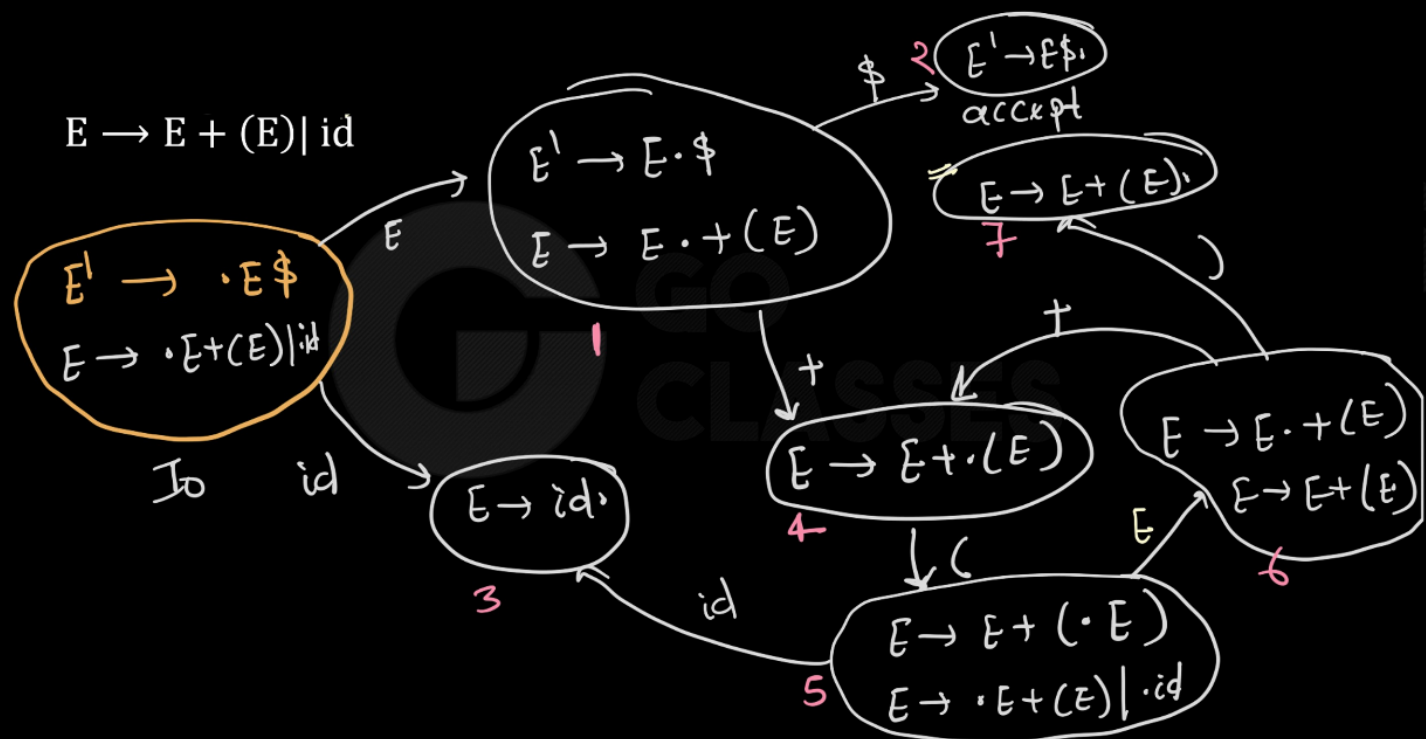
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## MSQ Question

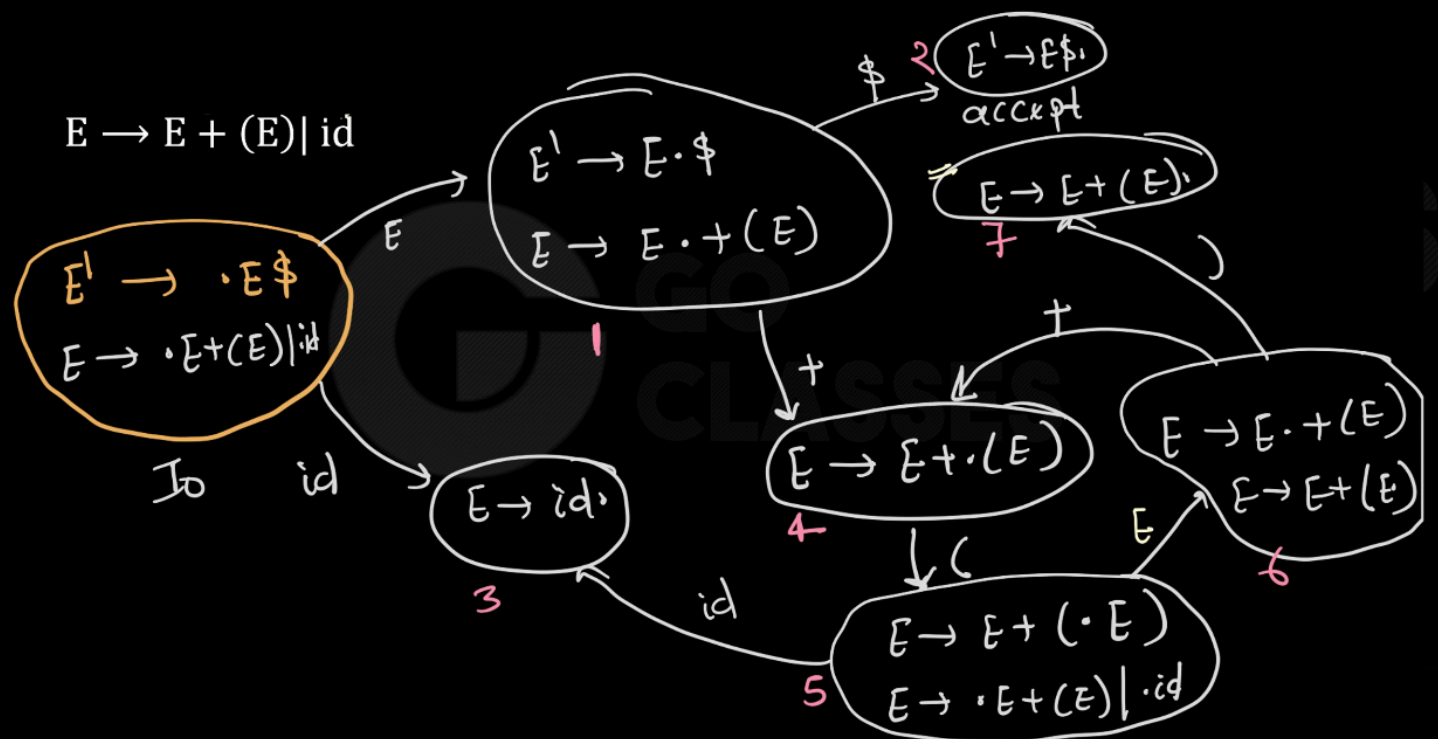
Consider the LR(0) automaton given for the following grammar.  
If parser is state 4 then what can we say about possible contents on the stack ?



- Stack contain could be "E+" where + is on top of stack
- Stack contain could be "id+" where + is on top of stack
- Stack contain could be E+(E+ where + is on top of stack
- Stack contain could be id+(id+ where + is on top of stack

## MSQ Question

Consider the LR(0) automaton given for the following grammar. Which of the followings are possible contents of stack in parsing ?



- a) id
- b) Id+ (here '+' is on top)
- c) E+( (here '(' is on top)
- d) E+E

## GATE CSE 2006 | Question: 7



Consider the following grammar

29



- $S \rightarrow S * E$
- $S \rightarrow E$
- $E \rightarrow F + E$
- $E \rightarrow F$
- $F \rightarrow id$

Consider the following LR(0) items corresponding to the grammar above

- $S \rightarrow S*.E$
- $E \rightarrow F.+E$
- $E \rightarrow F+.E$

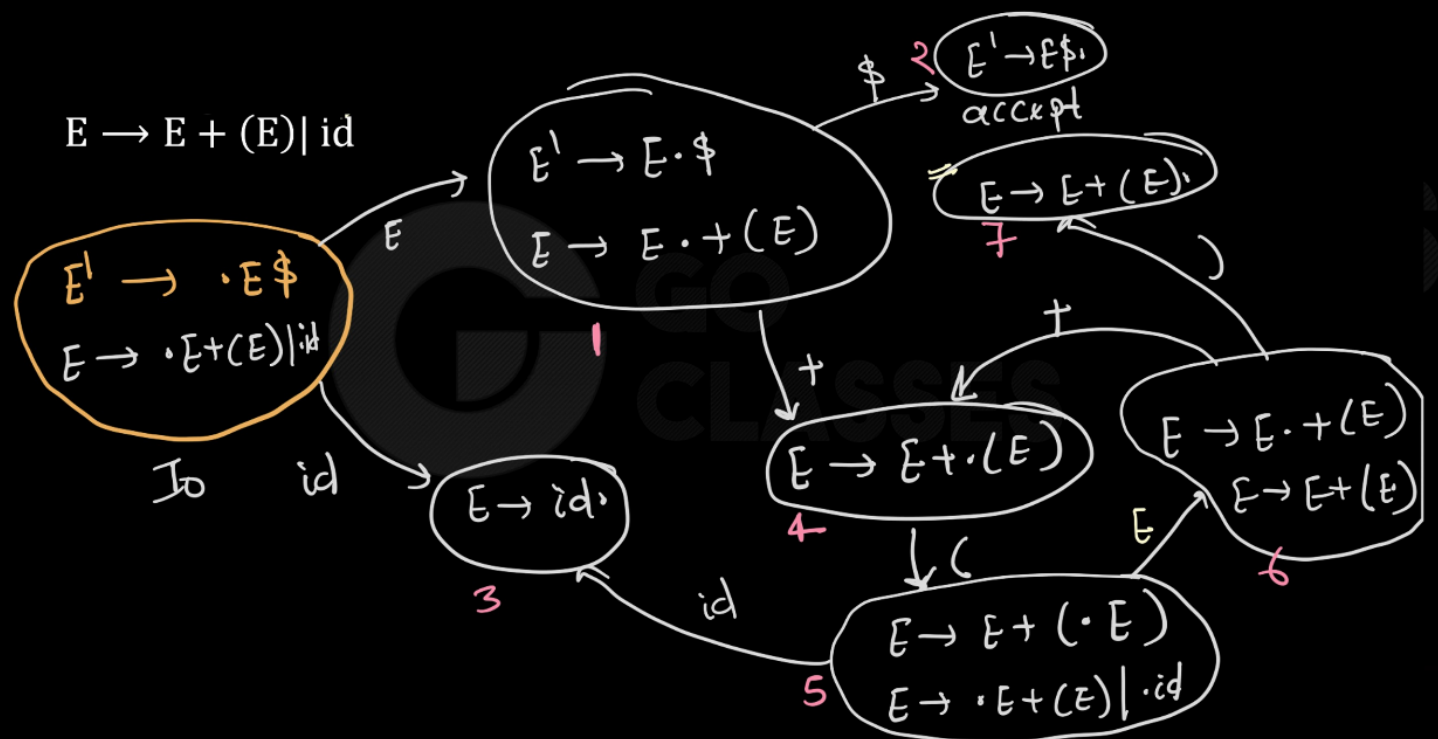
Given the items above, which two of them will appear in the same set in the canonical sets-of-items for the grammar?

- i and ii
- ii and iii
- i and iii
- ~~None of the above~~

- $S \rightarrow S*.E$
- $E \rightarrow F.+E$
- $E \rightarrow F+.E$

## MSQ Question

Consider the LR(0) automaton given for the following grammar. Which of the followings are possible contents of stack in parsing ?



- a) id
- b) Id+ (here '+' is on top)
- c) E+( here '(' is on top)
- d) E+E

~~id+~~

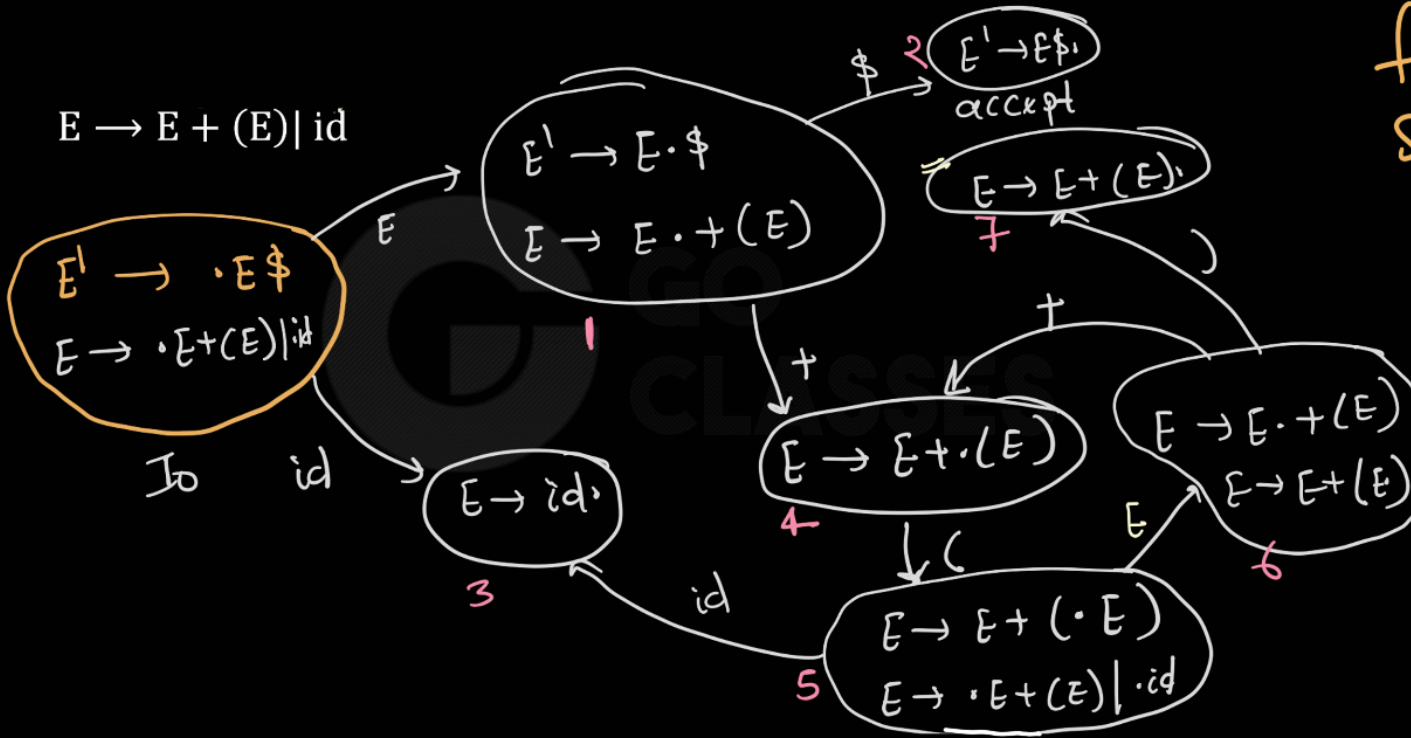


## Viable Prefix



Viabale Prefix

⇒ Content of stack



Which of the following can be stack content.

- $E$   ~~$(E \cdot )$~~
- $id$   ~~$(E \cdot ) E$~~
- $E +$
- $E + +$
- $E + ($
- $E + ( E$

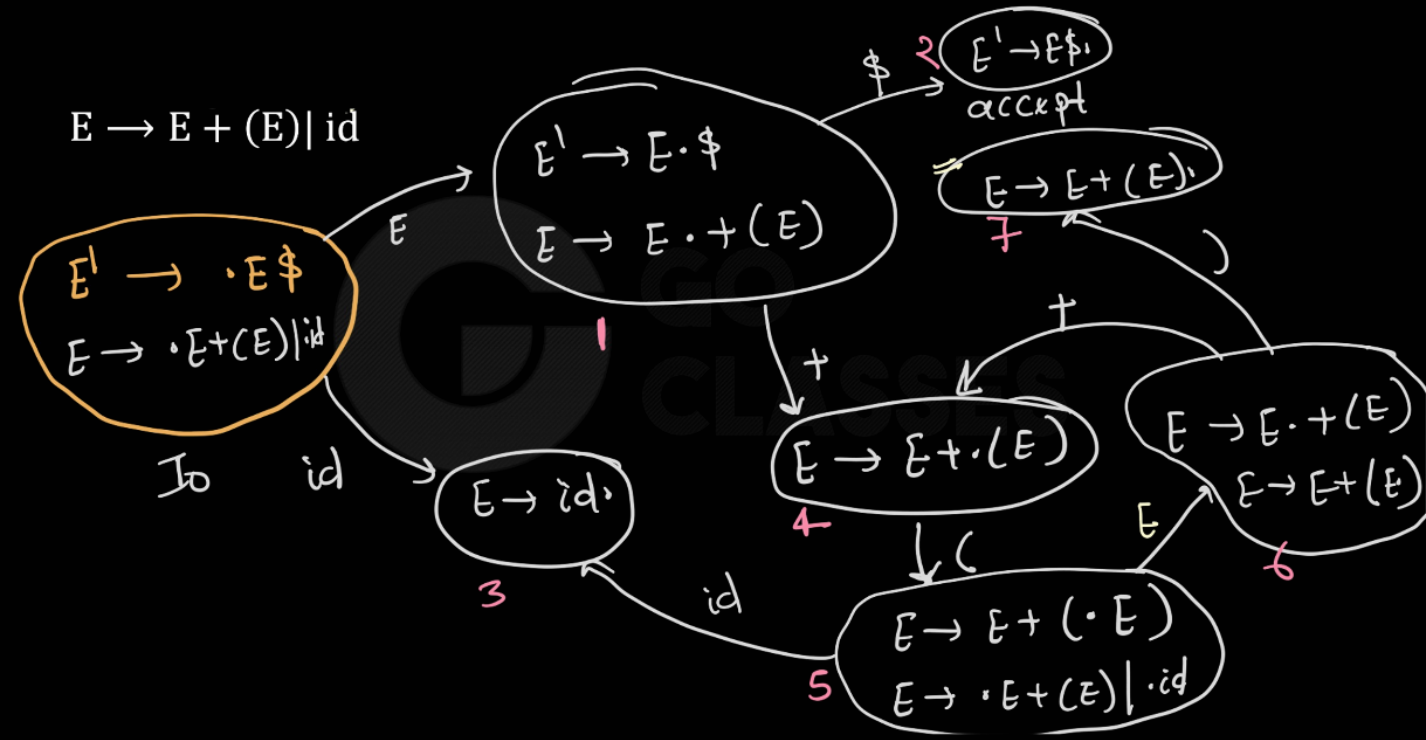
if some one ask you what can appear

in stack

⇒ we just need to check whether it is

leading to valid stack or not.

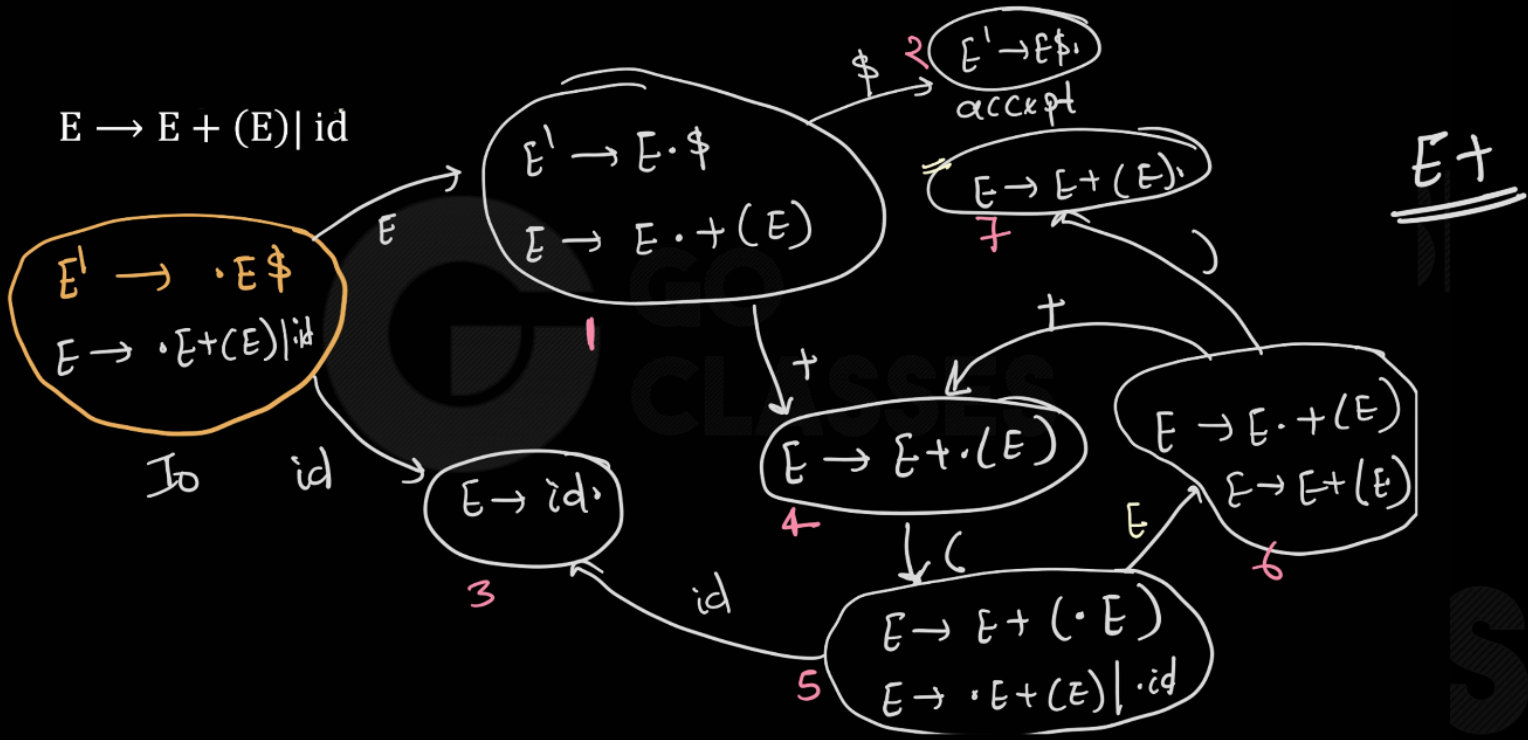
$E \rightarrow E + (E) | id$



which of the following can be the viable prefix

- a)  $E$
- b)  $id$
- c)  $id +$
- d)  $E +$
- e)  $E + E$
- f)  $E + ($
- g)  $+ ($
- h)  $id + ($
- i)  $E + ( E$

$E \rightarrow E + (E) | id$



Something that  
can be content  
of stack

which of the following can be the viable prefix

- ~~a) E~~
- ~~b) id~~
- ~~c) id +~~
- ~~d) E +~~
- ~~e) E + E~~
- ~~f) E + (~~
- ~~g) + (~~
- ~~h) id + (~~
- ~~i) E + ( E~~

mse  
Stack content is :  $\alpha \beta \gamma \delta$

then what are the possible viable prefix?

a)  $\alpha \beta \gamma \delta$

b)  $\beta \gamma$

c)  $\alpha \beta$

d)  $\alpha \beta \gamma$

mse

Stack content is :  $\alpha \beta \gamma \delta$

anything that  
↓ can be stack  
content

then what are the possible viable prefix?

~~a)  $\alpha \beta \gamma \delta$~~

~~c)  $\alpha \beta$~~

b)  $\beta \gamma$

~~d)  $\alpha \beta \gamma$~~



mse

Stack content is :

$\alpha \beta \gamma \delta$

anything that  
↓ can be stack  
content

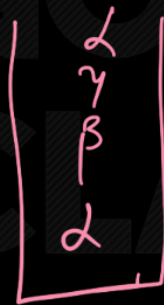
then what are the possible viable prefix?

~~a)  $\alpha \beta \gamma \delta$~~

b)  $\beta \gamma$

~~c)  $\alpha \beta$~~

d)  $\alpha \beta \gamma$

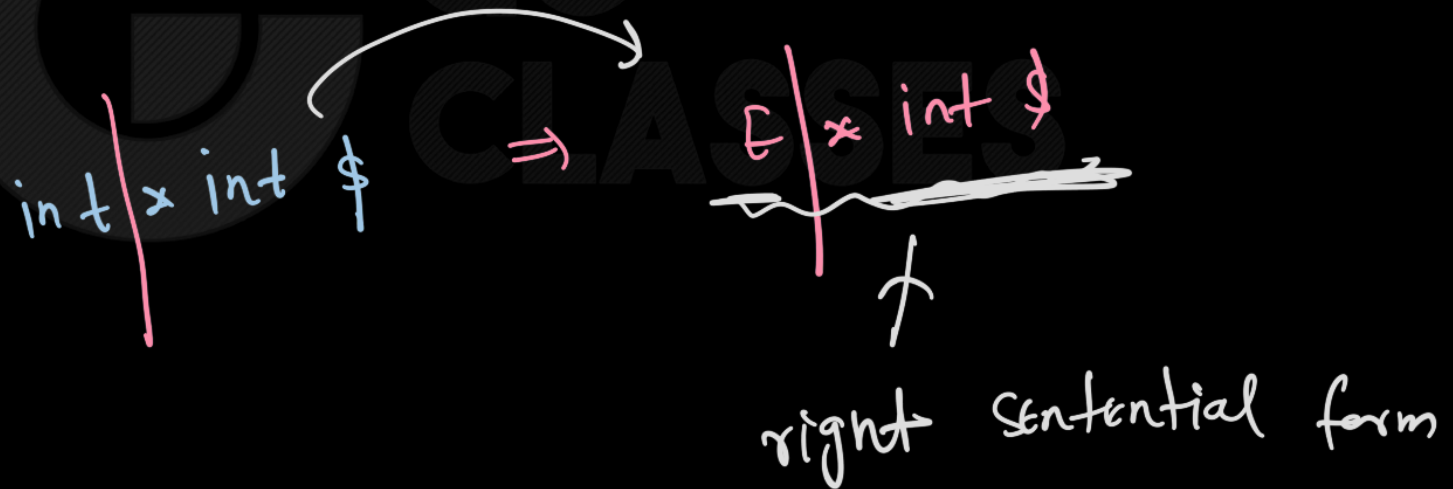


prefix of stack  
content was

the earlier stack  
content.



**Definition (viable prefix):** The prefixes of right sentential forms that can appear on the stack of a shift-reduce parser are called viable prefixes.





The set of viable prefixes correspond to the possible stack states in a shift-reduce parser.



<http://people.cs.uchicago.edu/~jhr/courses/w2004-51086/docs/notes-04.pdf>



## Ullman Question

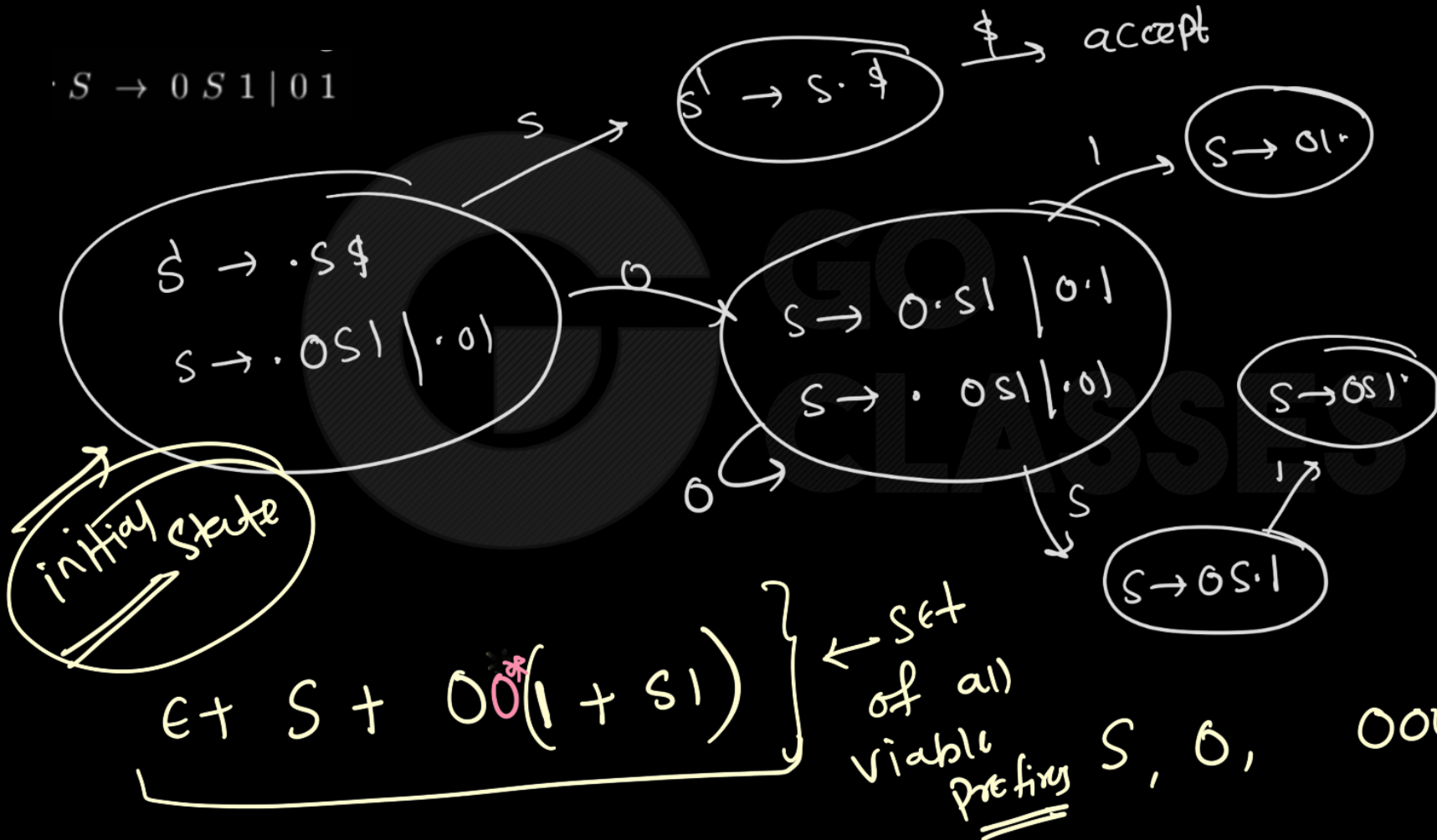
Exercise 4.6.1: Describe all the viable prefixes for the following grammars:

a) The grammar  $S \rightarrow 0 S 1 \mid 0 1$

b) The grammar  $S \rightarrow S S + \mid S S * \mid a$

c) The grammar  $S \rightarrow S ( S ) \mid \epsilon$

$S \rightarrow 0S1 \mid 01$



for viable prefix, treat every state as a final state



## True/False Question

---

The set of all viable prefixes forms a regular language





## True/False Question

---

The set of all viable prefixes forms a regular language

true

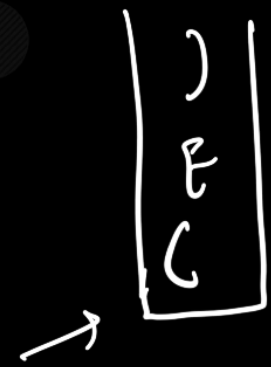
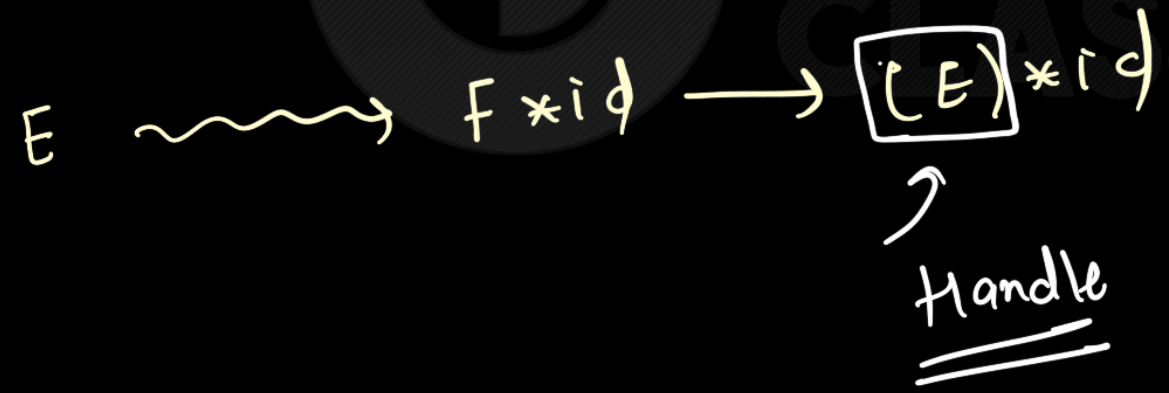
## Question

What are the viable prefixes of  $(E) * id$  in following rightmost derivation?

$$E \xRightarrow{rm} F * id \xRightarrow{rm} (E) * id$$

$(, (E, (E)$

any prefix of stack content



Stack content

## Question

Consider the given derivation, What is viable prefix of the right sentential form  $\alpha\beta z$  ?

$$S \Rightarrow^* \alpha A z \Rightarrow \alpha \beta z$$

- A. Any prefix of  $\beta$  is viable prefix
- B. Any prefix of  $\alpha\beta$  is viable prefix
- C. Any prefix of  $\alpha\beta z$  is viable prefix
- D. Any prefix of  $\beta z$  is viable prefix

$$S \rightsquigarrow \alpha A z \longrightarrow \alpha \boxed{\beta} z$$

$$\alpha \beta | z$$

$$\begin{array}{|c} \beta \\ \alpha \end{array}$$

stack content

## Question

Consider the given derivation, What is viable prefix of the right sentential form  $\alpha\beta z$  ?

$$S \Rightarrow^* \alpha A z \Rightarrow \alpha \beta z$$

- A. Any prefix of  $\beta$  is viable prefix
- B. Any prefix of  $\alpha\beta$  is viable prefix
- C. Any prefix of  $\alpha\beta z$  is viable prefix
- D. Any prefix of  $\beta z$  is viable prefix

$$S \rightsquigarrow \alpha A z \longrightarrow \alpha \boxed{\beta} z$$

$$\alpha \beta | z$$

$$\begin{array}{|c} \beta \\ \alpha \end{array}$$

stack content



## Question

- A. If  $\alpha$  is a viable prefix then we can always append set of terminals to get some right sentential form.
- B. If  $\alpha$  is a viable prefix then we can always append set of non-terminals to get some right sentential form .
- C. If  $\alpha$  is a viable prefix then we need to append mix of terminals and nonterminals (alone terminals or non terminals won't do) to get some right sentential form.
- D. If  $\alpha$  is a viable prefix then we can append mix of terminals and nonterminals to get some right sentential form.

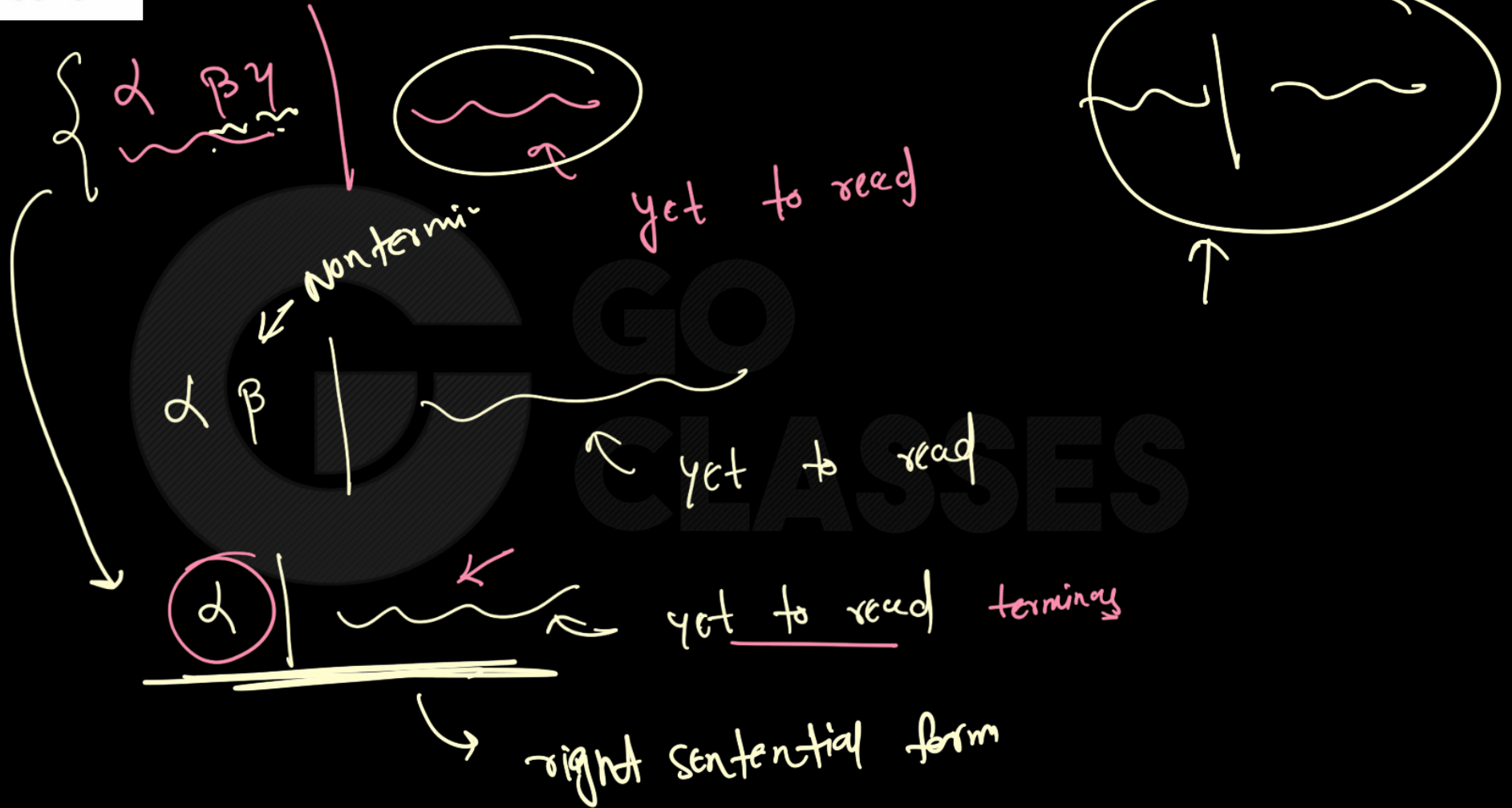




## Question

- A. ✓ If  $\alpha$  is a viable prefix then we can always append set of terminals to get some right sentential form.
- B. ✗ If  $\alpha$  is a viable prefix then we can always append set of non-terminals to get some right sentential form .
- C. ✗ If  $\alpha$  is a viable prefix then we need to append mix of terminals and nonterminals (alone terminals or non terminals won't do) to get some right sentential form.
- D. If  $\alpha$  is a viable prefix then we can append mix of terminals and nonterminals to get some right sentential form.









$\alpha$  is the only stack content



## Question

$$S \rightarrow id \mid S + S \mid id := S$$

Which of the following strings are viable prefixes? For each string that is a viable prefix, give a suffix that produces a right sentential form.

i.  $+ S +$

~~X~~ Not Viable

$$\begin{array}{|c} + \\ S \\ + \end{array}$$

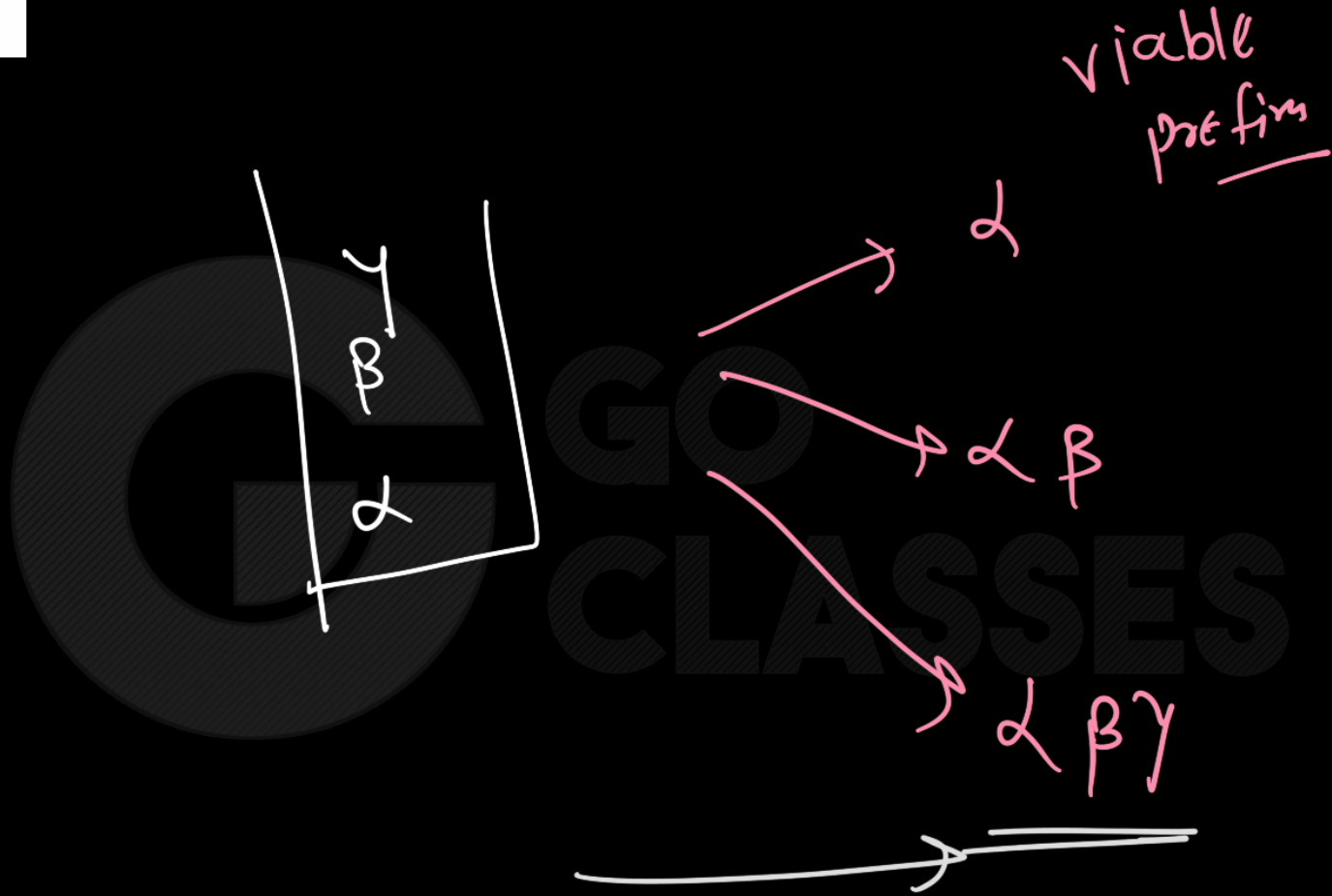
ii.  $S + id :=$

id

$S' \rightarrow \cdot S \$$   
 $S \rightarrow \cdot id$   
 $\rightarrow \cdot S + S$   
 $\rightarrow \cdot id := S$

+

<https://tbp.berkeley.edu/exams/2315/download/>



given this variable prefix, bottom of stack =  $\alpha, \alpha \beta, \alpha \beta \gamma$



# Answer

$S \rightarrow id \mid S + S \mid id := S$

Which of the following strings are viable prefixes? For each string that is a viable prefix, give a suffix that produces a right sentential form.

i.  $+ S +$

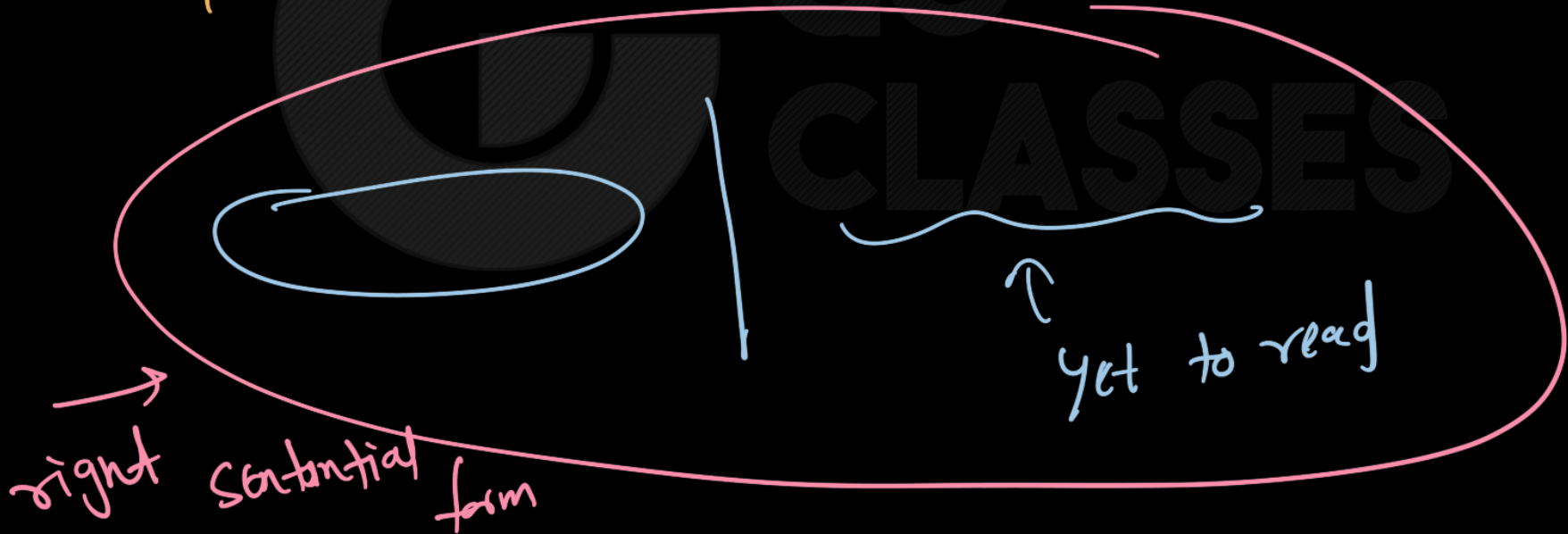
No.

ii.  $S + id :=$

Yes: suffix:  $id \$$

variable prefix

prefix of stack content is variable prefix



$\alpha$   $\beta\gamma$  | abc

$\alpha$  is a viable prefix what we need

to append to get right sentential form?

$\beta\gamma abc$

$\alpha$   $\beta \gamma$  | abc

$\alpha \beta$  is a viable prefix, what we need

to append to get right sentential form?

$\gamma abc$

$\alpha$   $\beta\gamma$  | abc

$\alpha\beta\gamma$  is a viable prefix, what we need

to append to get right sentential form?

abc

$\alpha \beta \gamma \mid abc$

abc

Right sentential form

viable prefixes

$\alpha, \alpha\beta, \alpha\beta\gamma$

RMP

$a \mid bc$   $\Rightarrow$   $A \mid bc$   $\Rightarrow$   $ABC \mid \$$

$AB \mid \$$

$S$   $\rightarrow$   $AB$   $\rightarrow$   $ABC$   $\rightarrow$   $Abc$   $\rightarrow$   $abc$

$\alpha \beta \gamma \mid abc$

Right sentential form

$(\alpha \beta \mid \$)$

viable prefixes

$\alpha$ ,  $\alpha\beta$ ,  $\alpha\beta\gamma$

~~$\alpha\beta\gamma a$~~

Possible handles

~~a)  $\beta$~~

~~c)  $\alpha\beta$~~

~~e)  $\beta\gamma$~~

~~b)  $\alpha$~~

~~d)  $\gamma$~~

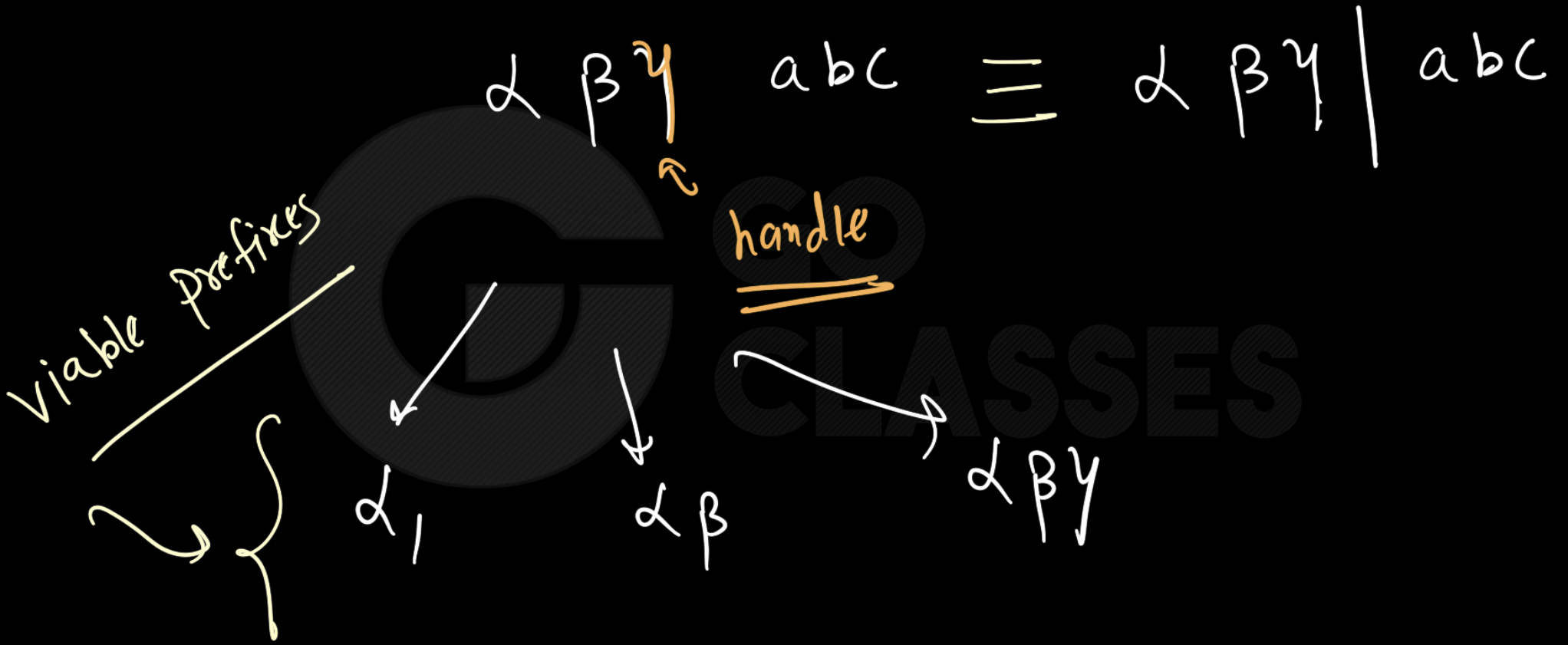
~~f)  $a$~~

$\leftarrow$  TOS

$\alpha \beta \gamma \mid abc$

$\alpha, \alpha\beta, \alpha\beta\gamma$

✓ viable prefixes



T/F

variable prefix is a prefix of any  
right sentential form where we do not go  
past by handle.

$S \rightarrow AB \rightarrow AbC \rightarrow Abc \rightarrow abc$

$\underline{abc} \$ \Rightarrow \underline{a} | bc \$ \Rightarrow \underline{Ab} | c \$ \Rightarrow$   
 $\underline{S} | \$ \Leftarrow \underline{AB} | \$ \Leftarrow \underline{AbC} | \$ \Leftarrow \underline{Abc} | \$$   
 $\underline{ab}$

$\underline{abc} \Leftarrow$  viable prefix  $\Rightarrow \underline{a}$

if  $w_3$  is handle then which of the following  
are viable prefix -

- $w_1$  ✓
- $w_1 w_2$  ✓
- $w_1 w_2 w_3$  ✓

$w_2 w_3$  ✗

$w_1 w_2 w_3 | w_4 w_5$

Set of all viable prefixes

⇓

draw LR(0) automaton  
and treat every state as final state  
initial state is fixed.

Given a string ⇒ what is viable prefix?

↳ locate the handle, put a bar and tell prefixes



viable prefix

← add some

terminals to get right  
sentential form



## GATE CSE 2015 Set 1 | Question: 13

Which one of the following is TRUE at any valid state in shift-reduce parsing?

- 52
- A. Viable prefixes appear only at the bottom of the stack and not inside
  - B. Viable prefixes appear only at the top of the stack and not inside
  - C. The stack contains only a set of viable prefixes
  - D. The stack never contains viable prefixes

gatecse-2015-set1

compiler-design

parsing

normal

viable-prefix

<https://gateoverflow.in/8187/gate-cse-2015-set-1-question-1-3>





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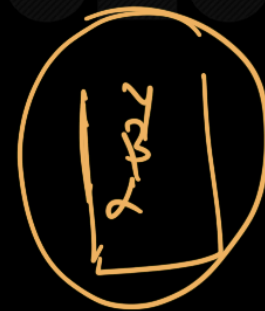
gatecse-2015-set1

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## True/False Question

Shift is allowed if after shifting, the content of STACK is still a viable prefix.

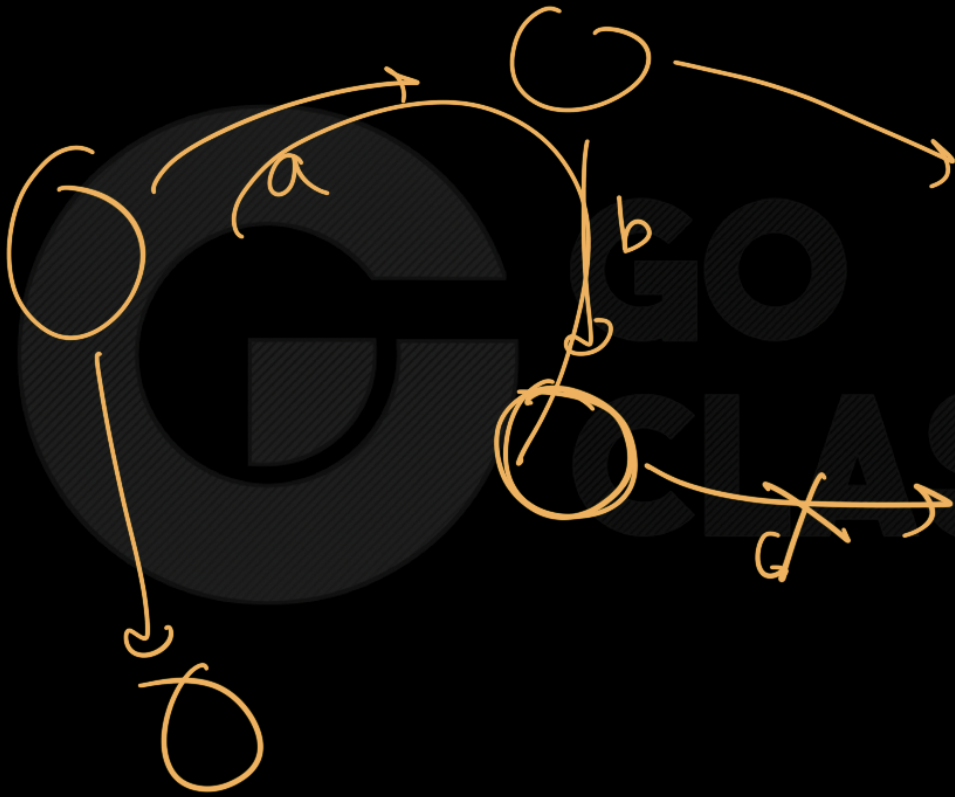




## True/False Question

Shift is allowed if after shifting, the content of STACK is still a viable prefix.

True

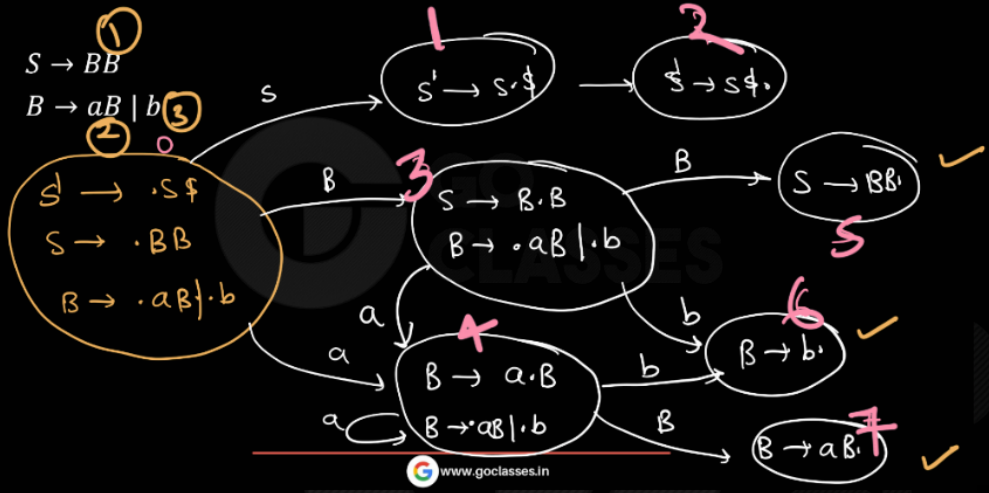


optimised version  
of rerun of DFA.

## Bottom-up parsing Algorithm



Stack contain  
State number on top



aabab

0 a | a b a b

a b c

0 B 3 | a b  
 ↓  
 0 B 3 a 4 | b  
 ↓  
 0 B 3 a 4 b 6 | \$  
 ↓  
 0 B 3 a 4 B 7 | \$

0 a 4 B 7 | a b  
 ↓  
 0 a 4 a 4 B 7 | a b  
 ↓  
 0 a 4 a 4 b 6 | a b

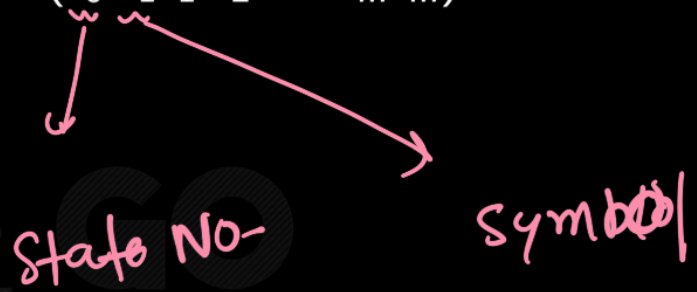
0 a 4 | a b a b  
 ↓  
 0 a 4 a | b a b  
 ↓  
 0 a 4 a 4 | b a b

⇒ 0 B 3 B 5 | \$ ⇒ 0 S 1 | \$

accept

Stack: contains configuration of the form  $(s_0 X_1 s_1 X_2 \dots X_m s_m)$

$X_i$ : grammar symbol,  $s_i$ : state



at Start :

Stack will have

0

State No.

0a1

→ re run of DFA

re run DFA

→ given any state of LR(0) automaton

we can tell possible stack contents

→ viable prefix

→ Bottom parsing algorithm (optimised version of re-run of DFA)